Mathematical Extracurricular Activities in Russia

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ABSTRACT

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The dissertation is devoted to the history and practice of extracurricular activities in mathematics in Russia. It investigates both the views expressed by mathematics educators concerning the aims and objectives of extracurricular activities, and the daily organization of such activities, including the pedagogical formats and the mathematical assignments and questions to which extracurricular activities have given rise. Thus the dissertation provides an overview of the history of extracurricular activities over the course of a century, as part of the general development of education (including mathematics education) in Russia.

The study called for a multifaceted investigation of surviving sources, which include practically all available textbooks and teaching manuals, scholarly articles on conducting extracurricular activities, magazine and newspaper articles on conducting extracurricular activities, surviving memoirs of participants and organizers of extracurricular activities, and much else, including methodological materials preserved in archives, which have been located by the author.

Summing up the results of the study, it may be said that two major goals have always been important in extracurricular activities in Russia: the first goal is motivating students; the second goal is preparing the mathematically strongest students and providing them with an opportunity to deepen and enrich their mathematical education. Of course, extracurricular activities have not been aimed exclusively at these two goals, and at different stages of

development additional goals (such as ideological preparation) were also formulated. Broadly speaking, it may be said that the history of the Russian system of mathematical extracurricular activities in general has been strongly aligned with the history of the development of the system of Russian school education. The study analyzes the specific character of extracurricular activities at each of the historical stages of Russia's development, in particular, it lists and described the basic forms of extracurricular activities, paying special attention to the indissoluble connection between the so-called mass-scale forms, in which millions of schoolchildren participate, and forms and activities that are engaged in only by a very few. Also provided is a survey of the changes that have occurred in the mathematical problems that are offered to students.

The author believes that familiarity with the longstanding tradition of extracurricular activities in mathematics in Russia may be useful also to the international sphere of mathematics educators, since the issue of motivating students is becoming increasingly important. The study concludes with a discussion of the possibilities and the expediency of putting such experience to use.

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Chapter I

INTRODUCTION

Need for the Study

The motivation of students to study advanced mathematics and the identification and development of their mathematical abilities are recognized as important tasks for the mathematics education community (Barbeau, Taylor, 2009; Leikin, Berman, Coichu, 2009; Saul, Assouline, Sheffield, 2010). Extracurricular activities, including all kinds of Olympiads and circles[1] (clubs), play a major role in this identification and development. Indeed, Olympiads prove to be an important means for the identification of prospective mathematicians. Their role has been investigated in many research publications. One of the directions of this research was studying the lives of former winners of Olympiads (Leder, 2011; Choi, 2009). These studies explored the process of forming mathematical interests of the future Olympiad winners, the influence of their teachers and coaches, and the later involvement of these winners in the organization and conducting of Olympiads.

It should be noted though that, first, Olympiads are by no means the only form of extracurricular activities, and, second, each country and even each region has its own. Russia (as well as the former Soviet Union) is among the countries where the educational system and practice of mathematics education attract international attention, and such attention is

[1] The Russian term “circle” became used internationally when related to mathematics education. Originally, this term related in Russia to any group of people working together on any theme. Saul and Fomin (2010) recall revolutionary circles where participants worked together on reading illegal literature, etc. Mathematics circles are typically groups of students that meet regularly and voluntarily work on mathematics.

understandable given that country prepared many winners of International Olympiads, and even more importantly, many first rate research mathematicians.

Russian mathematics education has been studied extensively through different viewpoints. Important contributions to these studies were the series of books edited by Izaak Wirszup and Jeremy Kilpatrick entitled *Soviet Studies in the Psychology of Learning and Teaching Mathematics* and the Vogeli (1968) text on schools with advanced courses of study in mathematics. Recently, the two-volume work by Karp and Vogeli (2010, 2011) explored many aspects of Russian mathematics education and its history.

Although the development of the Olympiads and approaches to their organization in Russia have been broadly discussed, other forms of the extracurricular activities popular in Russia are known but less researched internationally. Analysis of them is important for a better understanding of the entire structure of Russian mathematics education as well as the history of Russian education with all its approaches and patterns in its development. These approaches can be of help to mathematics educators in other countries. The chapter written by Marushina and Pratusevich (2011) attempts to outline some directions in studying Russian mathematics education. It seems important to develop this research further.

Purpose of the Study

The purpose of this study is to explore the history and practice of mathematical extracurricular activities in Russia. The study is guided by the following research questions:

1. What does Russian scientific literature say about the goals, objectives and forms of extracurricular activities in Russia?

2. What is the history of the development of the Russian system of extracurricular activities within the entire system of Russian mathematics education and, even broader, within the entire system of Russian school education?
3. What topics are typically covered and what problems are typically offered to students for mathematical extracurricular activities in Russia?
4. What are the most important forms of organization utilized in the system of Russian mathematical extracurricular activities?

Procedure

The study employs historical methodology and analysis of existing materials. To this end the important part of the research was devoted to identifying and collecting useful sources, including (but not limited to) research publications in mathematics education, manuals and teachers' guides, collections of problems, official documents, journalistic descriptions of existing practices, memoirs, interviews, etc. A systematic search in existing databases including both general and specialized catalogs (such as catalogs of research publications or dissertations) was conducted. Later, by comparing and contrasting identified sources, answers to the posed research questions were found. Specifically:

- To answer the first question, most important research publications (including doctoral dissertations) as well as practical publications (manuals and guides) were analyzed;
- To answer the second question, documents and publications of the time in question were analyzed; also, archival sources were utilized;
- To answer the third question, official state curricula as well as curricula of some specific educational institutions were consulted; additionally, notes and memoirs of teachers and former students (which give an idea of not only intended but also implemented curriculum of

studies) were examined; to analyze the mathematical problems offered in the extracurricular activities, collections of problems were explored and special coding was developed;

- To answer the fourth question, a similar approach was used – published and unpublished educational materials as well as some testimonies of teachers and former students were studied.

Outline of the Dissertation

The final report is planned to include 7 chapters, as well as an extensive list of references and sources. The following chapters will be included:

Chapter 1: Introduction

Chapter 2: Literature Review

Chapter 3: Methodology of Study

Chapter 4: Brief history of the Development of Mathematical Extracurricular Activities in Russia

Chapter 5: Programs, Forms and Practices of Mathematical Extracurricular Activities

Chapter 6: Problems Used in Some Mathematical Extracurricular Activities

Chapter 7: Summary and Conclusion

Chapter II

EXTRACURRICULAR ACTIVITIES IN RUSSIA: LITERATURE REVIEW

This chapter describes a few of the most significant and typical works devoted to extracurricular activities in mathematics in Russia (as well as the former USSR). It should be noted that because the literature on the system of mathematical extracurricular activities is very large, it is not possible to review all publications here. Moreover, the next chapters, particularly Chapter IV, which are devoted to the history of the development of this system will contain a literature review as well. This chapter focuses on scholarly papers and manuals that generalized practical or research findings (for example, those addressed to prospective teachers and introducing scientifically-based recommendations, even if only the authors of the manuals believed in the scholarly nature of these recommendations). Moreover, other directions in the literature will be briefly sketched here, while some will be explored further in other chapters.

Although some specific forms of extracurricular activities (mathematical circles or mathematical battles) are very popular outside of Russia and a few sources with mathematics problems for such activities have been translated, Russian literature on the subject of extracurricular activities is, in general, still virtually unknown internationally.

Among the sources in English one should mention the description of the Olympiad system in the USSR provided by Kukushkin (1996) as well as the chapter written by Karp (2009) which provides a brief history of Russian and American mathematically gifted education and describes some forms of extracurricular activities in Russia, comparing them with those in the United States. The chapter by Saul and Fomin (2010) in the two-volume set edited by Karp and Vogeli (2010) contains important information about Russian mathematics competitions and related literature. Another chapter in the same monograph written by Marushina and Pratusevich (2011) attempted to familiarize non-Russian readers with the existing Russian literature; however, only limited possibilities to do this were available within the framework of the book. This dissertation continues their attempt on a larger scale. Again, while it is not possible to discuss all books and papers published in Russia, the most important and typical works are considered in this dissertation.

Attempting to Analyze Theoretically

Books for Teachers

The book by Kolyagin et al. (1975) was written as a manual for prospective teachers of mathematics and has been used in this capacity for many years. This book includes a chapter on extracurricular activities in mathematics. First, the authors present an important discussion of two possible aspects of extracurricular activities. Defining extracurricular activities as voluntary, non-mandatory work which occurs before or after regular classes, Kolyagin et al. explain that these activities can be earmarked for those interested in mathematics as well as to those who struggle with it. According to Kolyagin et al., true extracurricular activities are only for those interested in mathematics. For the second group, who struggle, these additional classes should be offered only to students who miss regular classes for any reason. In other words, Kolyagin et al. believe that, ideally, *all* students attending the lessons should master the subject matter. The authors quickly caution that in practice, however, this is usually not the case, and teachers ought to invest their time in working with those students who are failing. Therefore, they offer some recommendations to this effect (i.e., maintaining a reasonably homogeneous group of students in these additional classes, meaning all students who have similar educational abilities).

Despite this distinction between the two groups of students, only those interested in mathematics are the real focus of Kolyagin et al.'s book (and specifically of this chapter). The authors suggested 10 goals of extracurricular activities, including the following four[2]:

- engaging children in doing mathematics and developing a stable interest in it;
- optimally developing children's mathematical abilities;
- developing the habit and ability to use popular and scholarly literature with children; and
- organizing an active group of children who are able to assist the teacher in producing effective work.

From this, it is obvious that the authors see extracurricular activities as being strongly connected with classroom activities. As for the content of the extracurricular activities, the authors underscore that although a few supplementary topics are to be covered during these activities, these topics should not be very different from the regular curriculum. Among the typical topics they listed were divisibility, certain geometric constructions (e.g., constructions with a straightedge only), and those from relatively recently developed areas of mathematics such as logic or combinatorics.

Kolyagin et al. also list a few different forms of extracurricular activities:

- mathematical circles;
- mathematical competitions and Olympiads;
- mathematical "events" (or festivals);
- supplementary reading—mathematical papers and essays; and
- school mathematical newsletters.

[2] All translations from Russian in this dissertation unless specially indicated are by the author.

The authors have discussed some of these forms, which are examined in greater detail later in this dissertation.

More recent publications directed at mathematics teacher education utilize an approach that is fairly similar to that taken by Kolyagin et al. (1975). For example, Sarantsev (2002) groups extracurricular activities in the same way—for those targeted as slow learners and those targeted as keenly interested in mathematics. As for activities aimed at the second group of students, Sarantsev categorizes them further into constant activities or those occurring at specific moments. The former category includes circles and school research societies which supposedly work year-round; the latter includes mathematical "events" or Olympiads. Sarantsev is a strong proponent of arranging "pairs"—a "lesson-plus-extracurricular activity" that establishes strong connections between regular lessons (mandatory for attendance) and an extracurricular activity attended only by those who are interested. According to Sarantsev, the most significant ideas represented during the lessons can be developed and extended during these extracurricular activities.

Among other books available for teachers, one cannot overlook Balk and Balk's (1971) manual. This differs from the Kolyagin et al.'s (1975) manual in that it not only discusses some theoretical issues and problems, but it also contains many specific assignments. The book has two parts: the first is devoted to methods of organizing extracurricular activities and their topics while the second provides actual material for this work. Regarding the form of extracurricular activities, Balk and Balk in general utilize the same classification that Kolyagin et al. employ, commenting on important methodological issues. One comment relates to students' possible misunderstanding of the mathematical circle's goals. That is, some students may think that the goal of this activity is simply entertainment because the teacher often uses puzzles and other "fun" activities.

A relatively recent book by Stepanov (1991) aligns with the books discussed above, but introduces some new perspectives. Specifically, the author emphasizes the importance of the preparatory step in extracurricular activities. Because participation in these activities is not mandatory, the teacher should identify ways of attracting students to them. Stepanov identifies two types of preparatory work: purely organizational and didactic. The first type should raise the students' interest in the activities (and, of course, these activities should be organized and managed in the most straightforward way). The second type, didactic, should assist students in overcoming any difficulties they face during the first sessions of the circle.

Another specific feature of Stepanov's book is its emphasis on paying special attention to the so-called "small-number-of-students schools," that is, those schools with very few students and typically located in small villages. Teachers in these schools face the problem of identifying topics for activities which may be of interest to students of different ages and levels of mathematical preparation. Also, the author mentions some useful characteristics of these schools—for example, these students typically have regular contact with nature and can explore it mathematically.

Another author, Kadyrov (1983), devotes his entire book to the theoretical study of the connections between extracurricular work (and specifically elective classes) and the regular classroom. The author underscores the role of continuity in education which he stated can be reached by having consistent goals in all forms of educational activities. In addition, Kadyrov explores ways of making extracurricular work more effective and emphasizes the following ideas:

- use of the history of mathematics in both classroom and extracurricular activities,

- use of elements of fun and entertainment when organizing educational activities, and
- construction of lessons as exploratory activities.

While more current books are usually the focus of research, older books should not be discounted for their importance in considering theoretical analysis. Berkutov's (1960) book was published far from the capital of the country but is still valuable in how it provides the opportunity to discuss the important role of mathematical extracurricular activities within a political and ideological framework (which was of critical importance in the USSR). The book starts with the text of the resolution of the Communist Party Central Committee entitled "On the tasks for the communist party propaganda in the current situation." According to the author, this resolution made raising the ideological and theoretical level of school mathematics education particularly important. In reality, despite this highly political beginning, the content of the book is fairly standard. The author discusses mathematical circles, events, meetings, and other routine activities. What makes the book special and less typical, however, is its attention to the historical perspective of the development of mathematics. While there is no time (according to the book) to study the history of mathematics systematically during lessons, it should be explored in detail after the regular classes. Berkutov's selection of recommended materials was clearly influenced by political considerations. For example, he recommends giving praise to the achievements of Russian mathematicians ("Abacus is a Russian invention"—a title like this is typical for a circle's meeting), describing the evil nature of the Christian sadists (illustrated by the story of Hypatia), and even telling "about the hard life of children in serfs-exploiting Russia"; he suggests illustrating the point with Bogdanov-Belsky's well-known picture, "Mental Computations in the School of S.A. Rachinsky." This task could be particularly challenging given that Rachinsky himself was depicted in the picture, a landowner-benefactor, who acted after the liberation of serfs. In any case, the material in the book was connected with mathematics and contained more than just political propaganda, as was often the case in other school subjects. For example, when different systems of measures were discussed, it was recommended that the topic of cheating be included to reflect on what merchants used to do before the revolution (p. 15); nonetheless, the measures themselves were still intended to be the center of the discussion.

Doctoral Dissertation Research Studies

Extracurricular work was also the focus of many dissertation studies. While this review mainly discusses the most recent ones, a few dissertations of an earlier period are mentioned in order to make comparisons.

Afonin's (1952) dissertation, defended in Uzbekistan, seems typical of its time, reflecting the period of the Cold War and the cosmopolitan campaign. For example, it specifically criticizes the American approach to extracurricular activities:

> The reactionary and lacking any scientific foundation style of the extracurricular activities (club work) in American schools: the leading principle of organization these activities aims to developing individualistic personalities, and competition among school students, and attempts to copy the adults' clubs. (p. 5)

Soviet schools, however, proved not quite free of problems, as the author explains; sometimes extracurricular activities were not sufficiently balanced, with too much attention paid to historical approaches and not enough time to problem solving. The author criticizes such unbalanced approaches and suggested his own (obviously well balanced) approach, materials for which constituted a major part of the dissertation.

Krolevets' (1956) offers an analysis of the history of extracurricular activities. According to the author, the entire Soviet period can be partitioned into three chronological stages:

- 1917-1933, “characterized as the time when practical forms of work developing practical skills and connected with the labor useful for the society were combined with the substantial development of mathematical extracurricular activities in yet prerevolutionary forms” (p. 3);
- 1933-1941, characterized by an increase in extracurricular activities of a formal-logical nature with less attention to practical skills; and
- 1945-1952, characterized by attention to practical skills and ideological-political development.

This dissertation is a rare example of a study of the history of mathematics extracurricular activities, which is the theme of the present study.

Another interesting feature of Krolevets’ study is that he discusses typical pedagogical mistakes in conducting mathematical circles. Among them are the following:

1. Homework is completed at meetings of the circles.
2. The curriculum of the circles simply repeats the standard school curriculum.
3. The curriculum of the circles is not connected with the school curriculum and is far removed from standard topics.
4. The curriculum of the circles is limited to the study of theoretical issues.
5. The entire work of the mathematical circle is devoted to solving puzzles. (p. 6)

Other forms of extracurricular activities are also analyzed in Krolevets’ dissertation.

Written two decades later, Mavasheva’s (1973) dissertation differs at least because another terminology is used: one of the chapters is devoted to experiments conducted by the author. These experiments deal with teaching that followed manuals which the author prepared. Specifically, the experiments address the following research questions: How should the sessions of the mathematical circles be structured? Should they be devoted to one topic and include information additional to the content offered during standard school lessons, or should each circle's session offer a diverse set of problems that are not thematically connected? The author believed in the first approach and her data confirm her opinion.

Although written 30 years after this dissertation, the work of Baisheva (2004) is structured in the similar way. She starts with a discussion of the importance of the Olympiad movement, noting the changes of the last decade when many new competitions emerged. She then sketches a new approach for the work in Grades 3 and 5 (based on using computers and other technology), and finally analyzes her experimental teaching which confirms (according to the author) the efficiency of her approach. Specifically, her approach suggests an organization of school competitions, including mathematical battles and Olympiads, by correspondence, the preparation of which is the most important practical contribution of the author.

Mardakhaeva (2001) seeks to analyze the system of work in mathematical circles for Grades 5-7 as well as the history of the creation of this system. Also, she attempts to discover how to prepare prospective mathematics teachers for this work. To address this issue and to improve students' preparation, Mardakhaeva offers a plan for a special course devoted to teaching mathematical circles. As a part of her study, she develops a few principles for organizing circle sessions, the syllabus for the circles, and materials for preservice students. These principles include the following:

- Principle of the connection between a school mathematical curriculum and extracurricular activities;
- Principle of the reasonable challenge of mathematics education in the system of extracurricular activities;

- Principle of a visual approach in mathematics education in the system of extracurricular activities;
- Principle of consistent developing student interest in mathematics in the system of extracurricular activities;
- Principle of inquiry-based education in mathematics in the system of extracurricular activities; and
- Principle of concentric education in the system of extracurricular activities.

Similarly, Demisenova (2004) devotes her entire work to the issue of teacher education and preparation for the teaching of mathematical circles. The goal of her study is to systematize and theoretically formulate the requirements for this preparation. Based on this theoretical framework, Demisenova attempts to develop a new course for prospective teachers, offering specific materials to support it. Based on her review of the existing literature, the author singles out several levels of teacher mastery in conducting extracurricular activities and identifies specific requirements for each level. For example, a student-teacher who reaches the first level is only able to develop a simple plan for the session and, with help, can identify tasks and supplemental literature for it; by contrast, a prospective teacher at the third level knows the existing literature very well.

Sergeev (2005) explores different aspects of extracurricular work. Given that the major educational tool of this work is mathematical problem solving, it is important to consider the most efficient ways of selecting problems for each circle session or competition. This leads to the issue of creating and developing a problem classification, which is one goal of his study. To accomplish this, Sergeev analyzes a number of existing sources, including collections of Olympiad problems, Internet resources, various magazines and books, and so on. In his work, the author faces some theoretical problems. For example, it is important to decide whether problems should be classified according to their mathematical content or according to the methods that can be employed for their solution. The notion of “method” also requires some theoretical discussion; indeed, the standard geometrical method of “doubling the median” can be thought of as belonging to the rubric based on the content—it deals with medians. Sergeev suggested a new approach based on creating a two-dimensional classification. This approach was used in the development of the website www.problems.ru which is still active.

Stukalova (2004) explores one additional aspect of extracurricular work. If Sergeev’s study is devoted to teaching very strong mathematics students, Stukalova is interested in those who are less talented but still college-bound. For college admission, these students needed additional assistance, and Stukalova is interested in developing a course of study specifically for them. She bases her suggestions on the systematic analysis of the difficulties experienced by college applicants when taking entrance examinations. She utilizes some new forms of analyzing achievement as well as some psychological theories.

Finally, a note should be made about the most recent work of Kochegarova (2013), which is devoted to the preparation of prospective teachers with what the author calls a *supplemental work.* This includes forms traditionally counted as extracurricular activities, such as Olympiads, circles, and the like. The author offers a theoretical model of the preparedness of teachers for this work, which is then tested by psychological and sociological methods using surveys and questionnaires. One critique of the study is that the author should pay more attention to the mathematical part of her study; however, her interest in non-mathematical issues such as psychological preparedness for teaching circles should be appreciated.

Literature on Specific Forms of Extracurricular Activities

The literature on specific forms of extracurricular activities is analyzed in detail in Chapters IV and V where the history of their development and forms is discussed. However, here in the general literature review, only broader comments are offered about the literature on the most important forms of extracurricular activities.

Mathematical Circles

The body of literature on the circles is considerable. There are circles of different levels (selective and mass), for different age groups (elementary, middle, and high school students), for different educational institutions, and so on. Correspondingly, all these differences are represented in the literature. Here, the review focuses on only the most important (most famous and often cited) and typical sources.

Among the most important books that must be considered are those published in the so-called Mathematical Circle Library (Shkliarskii & Chentsov, 1952, 1954, 1976; Shkliarskii, Chentsov, & Yaglom, 1962, 1974a, 1974b; Yaglom, 19551, 1955b; Yaglom & Boltyanskii, 1951; Yaglom & Yaglom, 1954). These books define the style and content of the teaching in the strongest and most selective mathematical circles of Russia.

Probably the best known book on mathematical circles outside of Russia was written by Fomin et al. (1994). This book was devoted to Leningrad circles, was translated into English, and has become the sourcebook for many circles and clubs outside of Russia. The book is a collection of problems, with some recommendations for teachers, and is suggested for use during the first two years of circle work; importantly, the authors assume that circles are very selective. In Leningrad practice, this course is usually covered by students in Grades 5-7 (the age when prospective winners of Olympiads usually start their studies). Formally, however, the book can be used for senior students as well. It includes sections on all typical Olympiad problems, including the Pigeonhole principle, divisibility, inequality, mathematical games, combinatorics, and so on.

The book by Gusev et al. (1977 was published almost 20 years earlier and is devoted to the same theme. Genkin et al. (1994) later commented that Gusev et al. had not covered some topics and, even more importantly, the difficulty of the tasks in this book increases too quickly. In other words, the work of Gusev et al. (1977) is written for another target audience. Hundreds of thousands of copies of the book were published and, obviously, the intended audience is not only the elite and most selective circles. At the beginning of each section, the problems are not very challenging; then, more difficult problems are offered to stimulate those who can do more.

Petrakov (1987) wrote his book for an entirely different audience again: high school students who were not interested in Olympiad mathematics but in a traditional and challenging version of school mathematics. While this book offers more difficult equations and inequalities than the standard textbook, solving them does not require the study of additional topics.

Books on Mathematical Olympiads

This field consists of a broad variety of publications and, above all, collections of problems. Because it is not possible to list them all, key groups of these books will be presented instead.

Petrakov (1982) attempts to describe the existing system of Olympiads. He comments that Olympiads serve to identify and develop mathematically-able students. Students who used to receive “threes” (Cs) and who suddenly discover at an Olympiad that they are successful in problem solving will (according to Petrakov) become engaged in studying mathematics more seriously. Also, he claims that Olympiads and, more generally, extracurricular activities are very useful for teacher development. These activities engage teachers in solving mathematical problems, reading professional literature, and so on. The book discusses all levels of Olympiads—school, district, region, all-Russia, all-Union, and international. Moreover, Petrakov explains the selection of problems for each level and provides relevant examples.

Farkov (2004, 2006) concentrates on the most mass Olympiads—school-level ones and formulates the following aims of school Olympiads:

- enrichment of students’ understanding of mathematics;
- development of their interest in mathematics; and
- identification of students for participation in the district (town) level of the Olympiads and individual work with such students. (Farkov, 2004, p. 4)

The book includes recommendations to the organizers of school Olympiads and to the students participating in them. It also provides examples of Olympiad problems.

Another book by Farkov (2006) is devoted to the school-level Olympiads of Grades 5-6. Interestingly, the author pays special attention to the Olympiads in small village schools. He rejects the practice of not conducting Olympiads in these schools (because of the small number of participants) and offers a new, more flexible approach to their organization. This approach involves using two parts—“standard” and “optional”—which can be accessible even to those who are less interested in mathematics.

Fomin (1994) and Leman (1965) are examples of authors whose books are devoted to higher level Olympiads. Galperin and Tolpygo (1986) are also important contributors to the field with a book that includes all problems offered at the Moscow Olympiads at all rounds. The authors discuss the issue of writing new problems and demonstrate that even during the years of the Cold War, the process of writing new problems knew no state borders. Some problems created in Russia were later published in a generalized form in America, and were then published in an even more generalized form in Russia again. Now, new collections of Olympiad problems are published relatively often (see, for example, Fedorov et al., 2008).

In addition to the "official" Olympiads included in the calendar of the Ministry of Education, many other Olympiads are held throughout the country. For example, major universities typically organize their own Olympiads. Typically, these Olympiads are competitions in solving challenging school problems. Guschin (2007) is an example of a publication devoted to those Olympiads.

Two additional books, which should be included in this discussion, are devoted to the Olympiads and are classics in this genre. In writing about the All-Union Olympiads, Vasiliev and Egorov (1988) analyze problems and described their organization, while Vasiliev et al. (1986) discuss the topic of Olympiads by correspondence.

While the books devoted to Olympiads usually are collections of problems, they often contain some general educational and historical information (which will be discussed later). Here it should be noted that certain episodes included in these books and saved for history are valuable: they help to characterize both gifted students as well as the organization and issues of the Olympiads. For example, Boltyansky and Yaglom (1965) share the interesting story of a student who became a winner in an Olympiad although he failed to solve any problem. Instead, this student concentrated on solving one relatively easy problem for which one had to make use of the "obvious" statement that no straight line can intersect all three sides of the triangle in its interior points. The student found that this statement was not obvious but required some proof. The Jury decided that this student demonstrated a very high level of mathematical maturity (indeed, the student himself realized the need for a special axiom which provides this statement). By contrast, such a situation could not occur in Leningrad: during the oral discussion, a student would understand that nobody expects such sophistication.

More recently, Tikhomirov (2006) recalls dramatic episodes in the history of extracurricular activities (specifically the Olympiads). In 1981-1992, the Moscow Mathematical Society was not permitted to participate in the organization of the Olympiads. Extracurricular work (just as any other type of work) proved to be part of political life in the USSR. Tikhomirov noted that these limitations eventually had positive outcomes because mathematicians sought alternative forms for educational activities and created the so-called “Tournaments of Towns.” This new competition (the actual creator of which was Nikolay Konstantinov) soon became international. Importantly, it combined competitive activities with other activities that were more usual for research mathematicians.

Literature on Reading by Students Themselves

Kolyagin et al. (1975) emphasized the importance of reading (independent study) by the students themselves; they emphasized, however, the methodological and psychological challenges facing the teacher in the organization of this work. Indeed, students are supposed to have enough will to work independently on reading books or writing relatively small papers. The methodology of this work seemed insufficiently developed in the Russian literature at the time of the paper’s writing, although it is particularly important now, given the many additional options provided by the Internet. Simultaneously, the popular literature available for independent reading is fairly well developed. One can mention, for example, all books under the title *Supplemental Pages for the Textbooks* (Depman & Vilenkin, 1989; Pichurin, 1999).

Conclusion

Even the limited set of sources analyzed in this chapter provided some insight into the diversity of approaches and goals of extracurricular work. Only the extracurricular work with the most mathematically gifted is well known outside of Russia, but that is just the tip of the iceberg. The literature on mass forms of extracurricular work clearly deserves international attention.

Chapter III

METHODOLOGY

This chapter describes the methodology of the planned study, which attempts to address the following research questions:

1. What does Russian scientific literature say about the goals, objectives, and forms of extracurricular activities in Russia?
2. What is the history of the development of the Russian system of extracurricular activities within the entire system of Russian mathematics education and, even broader, within the entire system of Russian school education?
3. What topics are typically covered and what problems are typically offered to students for mathematical extracurricular activities in Russia?
4. What are the most important forms of organization utilized in the system of Russian mathematical extracurricular activities?

General Principles and Considerations

Clearly, the posed questions assume the use of qualitative rather than quantitative methodology; more specifically, methods of historical research are mainly applied.

The methodology of the historical study in mathematics education was discussed in detail in Karp's (2014a) chapter and even earlier in Schubring's (1988) classic work. These writings emphasize the specific position of the history of mathematics education as a research field: while the history of mathematics education employs the historical-philological methods of study that are above all methods of analyzing historical sources that are usually texts, the texts themselves are often mathematical. As Karp (2014a) wrote:

> [W]e see the history of mathematics education as a part of social history, which becomes meaningful only when it includes social analysis and examines what happened in mathematics education in connection with the processes that were taking place in society around it. The development of mathematics teaching is an element of the social subsystem of schools, which, in turn, is a part of the broader subsystem of education, which interacts with other social subsystems such as the labor market, etc. (Schubring 2006). Mathematics education has appeared and developed not in isolation, but in response to various social needs. (p. 10)

It is hardly possible to expect that these social needs will be discovered and identified every time a mathematical problem or a form of the organization of extracurricular activity is discussed, but school mathematical life should be analyzed within the framework of the social history of the country at the time. This is the first and most important principle guiding the present study.

A social and historical framework provides a definite timeline and helps to identify the borders of meaningful periods of time that were explored in this study. Extracurricular activities are human activities, and although the lives of the people organizing these activities are discussed only marginally in this study, it is necessary to say from the outset that the conditions of their lives were important for their work. Karp (2010) related the sad life of a certain Soviet teacher in the 1930s who conducted a circle (albeit not mathematical) with sessions at which issues of interest to the children were discussed—namely, health, smoking, sexual life, and the like. Unfortunately, the teacher was accused of intentionally offering children activities that kept them away from political activities and serving the communist ideology. As a result, she was arrested by the political police. Clearly, conducting a mathematical circle could not result in anything similar (at least since 1930s). It would not be possible because at that time mathematics was viewed by the government as the most valuable school subject for defending the Motherland.

Another important consideration that instantly demonstrates how school extracurricular activities were located within the more general social framework can be found in the analysis of the position of mathematicians in the country. Many high-level research mathematicians were deeply involved in the organization of-extracurricular activities in the Soviet Union. The social and political environment was of great influence here. Indeed, Soviet mathematicians could hardly find any other social field except education in which to apply their social energy. At the same time, the mathematicians' participation in education was typically praised and supported. It is important to recall that the very important political campaign against the academician Luzin (Demidov & Levshin, 1999) was triggered by Luzin's critique in *Pravda*, the newspaper of the Central Committee of the Communist party, for unnecessarily praising a certain school he attended instead of criticizing the school for any issues and calling for its further improvement.

The second key principle guiding this study also comes from the analysis of social factors. The Soviet Union and even Russia are very diverse in their socioeconomic and cultural development. For example, some areas in Siberia are sparsely populated while Central Asia is overpopulated; also, lifestyle in Central Asia is very different from Siberian. These regions differ significantly from Moscow and St. Petersburg (Leningrad), which are huge cities with millions of highly educated citizens. Therefore, despite the role of ideological requirements (similar throughout the country), the differences in the books and materials offered to teachers in different parts of the country should be considered.

Before selecting the sources for the study, the researcher carried out substantial examination of catalogs in Russian libraries and archives. During the search, all types of publications devoted to extracurricular work were identified, including not only those published by the central (Moscow and Leningrad) publishers, but also those prepared by local professional development centers and publishers. A special role was assigned to the catalogs of the Russian National (former Public) Library, which is-currently the second in the country but was the first before the 1917 revolution. According to national law, this library received all national publications (although the extent to which this law has been recently enforced to include very small publishers is unclear). Thus, one may say that the selection of sources there was representative and complete. (Once the sources were identified, the copying of the existing materials was arranged. In that way, the author obtained her own copy of all analyzed sources including archival materials and could systematically analyze them).

The third key principle of the study relates to doing a careful exploration of differences (if any) between the descriptions given in the official literature and the reality of the classroom. With some reservations, all literature published in the USSR was official, expressed official requirements, and officially approved recommendations and good practices—otherwise, it could not be published. However, it is important to explore the relations between the intended and the enacted curriculum. (This juxtaposition exists and is explored not only in the USSR but in other countries as well, see Stein, Remillard, &. Smith, 2007.)

Clements and Ellerton (2015) believe that enacted (or implemented) curriculum can be seen from students' notebooks (many copies of which have survived from different epochs). For these authors, if a student wrote something in a notebook, it was most likely presented during the lessons. Unfortunately, notebooks do not permit identifying the place of copied materials in the curriculum. Moreover, many notebooks have undoubtedly been lost, and much information was never written down in notebooks. These and other points suggest that Clements and Ellerton's stand should be treated cautiously. It is clear, however, that the desire to understand what really happened during the process of teaching and learning as well as what the practice of teaching and learning was through student documentation is commendable (Ackerberg-Hastings, 2014).

In general, the present researcher based this dissertation on published materials, although in a few cases other sources were employed: memoirs, published interviews, and manuals were supplemented by some archival materials. Teachers' notes have very rarely been saved in the archive, but this researcher was fortunate to identify the personal records of one Moscow teacher which contain documents relevant for this study (these documents were copied in the archive for the author). Of course, materials submitted to the so-called district mathematical supervisor (which were found) are not absolutely objective descriptions of classroom reality. It is natural to assume that teachers corrected and added information to make this reality look better when reporting to their superiors. Still, it will not be an error to assume they did less of this work than was typical for official publication.

Memoirs and interviews are standard sources or instruments for historical study (Karp, 2007, 2014). The number of these sources relevant to the present study and available to the researcher was limited. Still, use of these sources seems important because they represent another perspective—specifically the students' perspective—thus making this study more objective and fuller.

In sum, the major approach employed in this study is a historical approach, one which compares and contrasts different sources and thus employs a critical yet respectful attitude towards these sources.

The Elements of the Quantitative Approach in This Study

Mention was made of a specific feature of historical studies in mathematics education—that the scholar should work with mathematical texts. This study faced all related issues because one of its goals was to explore mathematical problems offered in different extracurricular activities.

Many approaches have been offered for classifying mathematical problems. One leading principle employed for classification was the problems' role in the teaching and learning process, other were the complexity of problems, the scheme of the solution, and so on (given that Russian education is being discussed, reference for the different approaches to the problems' classification should be given to Russian publications [Sarantsev, 1995; Stolyar, 1974]). This study is limited to the discussion of the themes of problems, but even this approach was not easy methodologically. Although analysis of problem themes is standard procedure in mathematics education research, it can be criticized as insufficiently accurate. Below is one example demonstrating the possible challenges inherent in this approach.

Consider the following randomly selected Olympiad problem (offered in St. Petersburg at the so-called *district round* in 1996; see Berlov et al., 1998, p. 63):

> The polynomial $x^3 + ax^2 + 17x + 3b$, where a and b are integers, has three integral roots. Prove that they are distinct.

The solution provided by the book's authors suggests the following steps:

1. Let's note that the product of all three roots x_1, x_2, x_3 equals *3b.*

2. It implies that at least one of these three roots is a multiple of 3, and for instance, the root x_1 can be expressed as 3s for some integer s.

3. It follows (from the so-called Vieta theorem) that $3sx_2+3sx_3+x_2x_3=17$.

4. Then the product x_2x_3 has the same remainder when divided by 3 as 17 (that is 2).

5. But if the product of two integers has a remainder 2 when divided by 3, then none of them is a multiple of 3, and they have different remainders when divided by 3.

5. Now the conclusion can be made that the integers x_2 and x_3 are distinct (given that they have different remainders) and the number x_1 is not equal to any of them because this number is a multiple of 3.

As indicated, this proof employs a knowledge of the algebra of polynomials as well as elementary number theory. The question thus raised is: To which subject area does the problem belong?

Clearly, other solutions may be attempted—for example, the derivative of the given polynomial can be considered. However, here, another methodological predicament is faced: this problem was offered to 10th graders, but in 1996 most of the students studied differential calculus in Grade 11 only. The Vieta theorem for cubic polynomials was not studied in regular schools either, while in classes with an advanced course of study in mathematics, both the derivative and Vieta theorem were covered in Grade 10 (Karp & Nekrasov, 2000).

In sum, even a discussion of problem theme and its correspondence with school curriculum is a challenge. Generally speaking, students who never heard of the Vieta theorem for the third degree polynomial can invent its formulas themselves and in this way show that the task is in the standard curriculum.

It is hardly possible to identify one single method of classification that is free of any deficiency. All calculations[3] about problems from Russia, the United States, or any country for that matter can be criticized—and actually have been. -Thus, it seems appropriate to agree from the outset that in the field of classification problems, all calculations are only approximate estimations. This approach is not unusual; in humanities, for example the concept of the theme of a poem is discussed and relevant calculations are sometimes done (see, for example, Gasparov, 2001), although the limitations of these calculations are clear. Consequently, the following approaches were used in this study.

First, problems were assigned to groups according to the object dealt with in the problem, rather than according to its solution. For example, the problem discussed above was treated as a problem about a polynomial and, therefore, is an algebra problem rather than a theory of numbers problem or a calculus problem, whatever solution can be given.

Second, when identifying themes, only very large themes were considered, such as the names of school mathematical subjects—algebra, trigonometry, plane geometry, solid geometry, numbers, and operations. Even a preliminary analysis of Olympiad problems suggested singling out one more group which (to some extent, following Russian tradition) is called *constructions*. Often, these constructions are borrowed from discrete mathematics, the borders of which are not often rigidly defined. These problems can be called *word problems* (and typically they are offered in the form of some narrative), but this term was reserved for one more group dealing with traditional problems (both in the form and methods of solutions).

[3] Here and below, we employ the word "calculations," which corresponds roughly to the Russian word "*podschety*," used in Russian philological studies, which to a certain extent serve as our model. Broadly speaking, the term refers to attributing a problem (or some other text) to one or another group, and subsequently identifying the number of items belonging to this group, their share in the overall corpus of texts, and so on.

By contrast, construction problems typically deal with the objects that are not studied in a school course.

It is worth emphasizing again that this approach to identify a problem's theme is not absolutely rigorous, and minor differences in judgment by different people seem to be unavoidable. The role of these judgments is very different from the role of calculations in statistical studies. Here, they serve as an illustration of the existence of different themes and changes within these themes, which are visible enough to permit the use of this method, despite its deficiencies.

The methodological foundation for this part of this study was based on a paper by Karp (2015) who analyzed the distribution of problems in Russian textbooks. All calculations there were followed by examples which provided an understanding of how and why the problems were assigned to this or that group. Again, this approach (recalling studies in the humanities) is possible only if the processes under investigation are visible and the combination of calculations and description is therefore, persuading. For example, Karp demonstrated that the number of *preparatory* problems in textbooks (that is, problems preparing students to understand concepts or algorithms) at some point increased dramatically; they were hardly represented in the textbooks of the previous generation. Thus, it seems that the changes in the content of the Olympiad problems are even more visible, and, therefore, a similar approach can be used in this study.

Chapter IV

HISTORY OF THE DEVELOPMENT OF MATHEMATICS EXTRACURRICULAR ACTIVITIES IN RUSSIA

This chapter addresses the research question: *What is the history of the development of the Russian system of extracurricular activities within the entire system of Russian mathematics education and, more broadly, within the entire system of Russian school education?* The narrative of this history is structured in chronological order.

Mathematics Extracurricular Activities Before the Revolution of 1917

It is hardly possible to identify one single date as the beginning of mathematical extracurricular activities in Russian schools. However, what can be stated is that, on the one hand, these activities were clearly less popular in pre-revolutionary schools than became later and, on the other hand, they still existed in schools well before the revolution.

The Russian translation of *Récréations mathématiques* by Lucas (1883) revealed that even many years ago, there was a need for " the sophisticated questions, fun and games" (to cite the sub-heading of the book), which is something that often serves as a foundation or at least as a substantial part of mathematical extracurricular activities. At the First Congress of Russian Mathematics Teachers, a special presentation was made about the role of games and other activities "serving the development of visual thinking and representation" (Smirnov, 1913). Discussed in the presentation were not so much extracurricular activities as we understand them today, but rather a form of manual labor as a means of developing mathematical thinking (which can be considered a special form of mathematical extracurricular activity).

Popular mathematical journals such as *Zhurnal elementarnoy matematiki* or *Vestnik opytnoy fiziki i elementarnoy matematiki*, which replaced it, did not discuss extracurricular activities explicitly, but they did address students who sought mathematical work beyond the typical classroom. In this way, they served as a manual for extracurricular work in any form. The editorial preface to the first issue of the journal *Vestnik opytnoy fiziki i elementarnoy matematiki* in 1886 stated:

> Our Journal is intended first of all for young readers who are educated in our institutions. It will strive to address the desire of the broadening the perspective in the area of physics and mathematics sciences; that is the desire which is particularly strong in young age, and which manifests itself among students in a form of a very strong attempt to learn more that it is assigned according to the official curriculum. (p. 1)

Fridenberg's (1915) report at the Second Congress of Russian Mathematics Teachers can be considered an important step; it was cited many times later (including during the period after the Revolution) and already contained the standard term in use now ("extracurricular activity"), as well as provided some observations and recommendations for the organization of these activities. The presenter emphasized the importance of the individual approach and criticized the standard classroom routine:

> We assume that students need to act as a choir and this approach in teaching ignores all individual requirements of students, and destroys their personality by accommodating it to the average level of the class. It doesn't permit any creativity towards the subject. The form of the class itself, these "official" 45-50 minutes which artificially limited all the students' work, do not allow any space for the interest and passion. (p. 145)

He continued:

> To satisfy the individual interests of students and to develop their creative ability we need to organize systematically scientific circles stating from the junior school through the entire school course. The style of the circle sessions is very different from the standard classroom. No detailed syllabus is in need here, instead, a general plan and

> clearly stated goals would suffice. These circle meetings are not mandatory. Any child—boy or girl—selects his or her work according to the interest and ability and they work not in a rush, not even thinking about the time. (p. 145)

It proved necessary, however, for Fridenberg to note that in practice not everything goes as smoothly as intended. Specifically, overloading students with regular school work as well as issues in planning classroom and out of-classroom activities were found to pose many obstacles. Still, the presenter reported about his success in organizing circles in one real school (*realschule*) and in one gymnasium in Moscow. The sessions were conducted in the evening (6:00 p.m.-10:00 p.m.), and about 30-60% of the class usually opted to participate in them. The curricula of junior classes included preparation of models or (as the presenter put it) "independent work in some part of the mathematics" (p. 146). Additionally, the students in these classes were very fond of puzzles and logical problems (p. 147). In senior classes projects on historical and philosophical topics dominated. Importantly, the results in conjunction with the preparation of the manuscript were reported publicly (and everybody was welcome to attend these presentations). Among the popular topics, the following were listed: Pythagorean union, history of elementary geometry in ancient times, different systems of numeration used in history by different nations, and the possibility of different geometries. Additionally, attention was paid to applications. For example, a plan (map) of the environment of the local lake was prepared. The report emphasized the particular importance of collaboration among different teachers in this integrative activity—specifically, mathematics teachers, science teachers, and even teachers of Russian.

The present researcher managed to discover in the collection of the Russian National Library in St. Petersburg some materials which provide additional evidence that mathematics circles in fact existed before the Revolution of 1917 as well as details about their work. Since the end of 1905, there was a mathematical circle at the Orenburg Real School (its original name was

“Mathematical Evenings for the Students”). The major organizer of the circle was a teacher of mathematics, N.I. Shemianov. In 1906, the circle’s participants began publishing a student journal entitled *Notes of the Mathematical Circle at the Orenburg Real School* (*Zapiski matematicheskogo kruzhka pri Orenburgskom real’nom uchilische*), which, as mentioned earlier, survived in the Russian National Library. The 1911-1912 issue contained the note “Some information about the circle” which summarized its activity.

The creation of the circle, not to mention the publication of the journal, was-possible only after the permission from the supervisor of the Orenburg educational region was obtained in February of 1906. The chair of the board of the circle eventually became the director of the school. Until 1911, the circle had no systematic code of rules, and it was only in 1911-1912 that the Pedagogical Council, “concerned with the fate of the mathematical circle” (which indeed practically did not function two years in a row before), provided special guidance (p. 6). It was specially explained that membership in the circle could be granted to the teachers and students of senior classes only, while students could not vote in the election of the circle’s board. A summary in the journal reported that at the moment of its publication, 15 teachers and 45 students were among its members. Non-members were not allowed to participate in the meetings of the circle (which automatically excluded any external political propaganda); however, it was also noted at that point that this rule was not always followed.

At these sessions, members gave presentations and also solved “the most interesting problems” (p. 6). To make a presentation, a member had to obtain approval of the manuscript by the secretary of the board (a mathematics teacher). There was a fee for membership in the circle, although it was only “symbolic” for the students—one ruble per year, which covered receiving the circle’s publications and the privilege of using the circle’s library. The library offered 48 publications. Some of them were written in Russian, some were translated into Russian, and some were in foreign languages.

Students' presentations were devoted to a variety of themes, including "Stewart's Theorem and a few geometrical problems," "Division of polynomials and extracting roots with the method of undetermined coefficients," "Proof of some properties of trapezoids utilizing arithmetic progression formulas," and so on. Accordingly, the journal published solutions for the offered problems, reviews of new courses, and critical comments on the new manuals (written by the teachers).

Although only a few examples of circles were considered, there is no doubt that there were a relatively large number of circles in the country. It is important to keep in mind, however, that even a standard secondary education was available only for a select few, while the substantial part of the population received no education at all (estimates here vary, but even the most optimistic expert, former minister of education Ignatiev [1933], thought that in 1917 approximately one out of 10 children did not attend school). Given this context, extracurricular activities in mathematics could be considered rare in the country.

Mathematics Extracurricular Activities in 1917-1956

If the first date in the heading—1917—can meet no objection, given that the October revolution completely changed Russian life in general and Russian education in particular, the second date—1956, the year of the twentieth Congress of the Communist party at which the so-called cult of Stalin's personality was criticized—is not as exact. This year indeed initiated the *unfreezing* of social life (it was called a *thaw* following the popular novel of the time). Still, there were several other very important dates that could be considered the end point of this period.

Another option is to subdivide this period, the first part being 1917-1931 and the second 1931-1956. In this case, the year 1931 is a benchmark because it was the year of the first Communist party Central Committee resolution on education (Karp, 2010). Although this date and corresponding changes will be discussed in the following sections, the larger period is considered a united time to some extent. Educational processes were not quick to develop: the counter-reforms of the 1930s were greatly influential in general and in the history of extracurricular activities, in particular, however, some of their results appeared only later and over time. Therefore, it is more useful not to interrupt the historical narrative by subdividing the time period.

After 1917

It is worth repeating that the 1917 Revolution changed education fundamentally. The system in which simultaneously coexisted elite gymnasia, village schools, and many types of schools and institutions between these two was destroyed momentarily (Karp, 2010). Instead, unified labor schools of the first and second stages were created. Mathematics, which had been treated as the queen of school subjects in Russia since the 18th century (Karp, 2014), suddenly became a service course only studied for its useful practical applications. Thus, nothing had to be studied that did not serve this goal (e.g. developing deductive reasoning became unimportant). Moreover, an axiomatic organization of the course could not be offered anymore. To some extent, the extracurricular activities proved to be more important than the standard curriculum. Mathematics had been taught not as a separate school subject, but as a part of a so-called *complex* (that is, an object studied from different perspectives and using methods from different research areas).

For example, one recommendation was to study the concept of the function by observing during an outdoor excursion how agricultural technique works. Karp (2012) cited the paper of the teacher Nazarenko, who described how students visited a state farm attached to a factory:

> The field trip yielded much material: “The first thing that we did, en route to the state farm, was to determine distances by eye and then immediately verify them by using a tape measure, counting steps” (p. 62). Subsequently, things got even better: students noted the sizes of haystacks, computed their volumes and the weight of grain, using reference data provided by an agriculturist. “Rich material was yielded by the threshing machine”: here, the students were able to compute speeds and obtain a visual notion of straight lines and inverse proportionality, as well as a whole series of statistical data. Moreover, they were able to obtain a “vivid notion of functional dependency—the pulley of the electric motor, the rotation and motion of the threshing machine’s parts.” (p. 63)

Excursions were very popular in mathematics teaching. Shemyanov’s (1925) booklet was devoted to this approach. He claimed that excursions provided many opportunities to demonstrate the real-world applications of mathematics and useful mathematical connections with other areas of knowledge. Interestingly, the author believed in using special mathematical excursions in which all attention is focused exclusively on mathematics (in contrast to claims of the dominating *complex* method that the school should teach all subjects in a complex). As an example, he recommended excursions during which students measured the heights of trees (utilizing similarity) or prepared maps (utilizing ratios), among other things.

Another example of a book devoted to the same theme is by Minervin (1927). This book, however, contains fewer theoretical discussions and many more tasks for excursions. In fact, the book is a collection of practical tasks that can be offered during excursions (or just outside the classroom). The very first problem provides a good example: “Find the length of your own step.” To solve this problem, Minervin suggests measuring how many steps one needs to cover 20 meters. Students are then advised to divide 20 meters by the obtained number and determine the length of the step. The author recommends repeating this procedure 10 times and finding the mean in order to minimize any error in measuring.

This problem is accessible for elementary school students; other problems in the book are much more challenging. One, for example, requires finding the depth of a hole by connecting its deepest point and its surface with a rope, then measuring the length of the rope and the angle this rope forms with the surface (trigonometry should be used here). A sequence of tasks under the title “Excursions to a river” include measuring the width of a river or some other distances that cannot be measured straightforwardly.

Excursions, laboratory work (including that conducted outside the school), and different projects—all these forms were popular and attempts were made to popularize them even further (Zaks, 1930), although their value from a traditional perspective was not always substantial. Among these new forms, however, there was a place for the mathematical circles that had come into existence earlier.

Sigov (1924) explains that school “should help a student to find the answers to the questions which he himself could ask, and should help this student to enrich and deepen his knowledge if he wants to do it” (p. 11). Because post-revolutionary schools were afraid of any inequity and pressure, Sigov tried to avoid any misunderstanding and, therefore,. felt it was necessary to add that “classroom studies by no means will suffer for students if these students will choose not to attend the circle; the themes for the circle’s sessions should be selected in such a way as to avoid any visible connection with the issues discussed in a regular classroom” (p. 11). The *invisible* connection Sigov is implying was not discussed. Instead, he explained how to organize a circle, how to select topics for the students’ work, and how to distribute these topics among them. Typically, these topics were historical: “How did ancient people calculate?,” “Mathematics in India,” “Logarithms and their invention,” “Lobachevsky and his geometry,” and so on. Books and manuals supported by the Ministry of Education (Narkompros) were listed and recommended for study.

Moreover, independent mathematical investigations were recommended. Specifically, Sigov recommended reading any “popular and clearly written booklet, say, about some industry or some processes in this industry” (p. 12) and “listing all the issues which can be explored from the mathematical perspective. At that point Sigov gave a further recommendation not to waste time on reading clearly written booklets, but rather to visit any factory—an activity that would be far more thought-provoking. This would lead to opportunities for all kinds of calculations for, say, the speed of rotation of specific details or their ratio. Even more options can be obtained by randomly selecting any technical journal. Sigov himself randomly selected the journal *Vestnik metallurga* (*Metal Worker’s Newsletter*) of 1922 and the paper “Organization of the Work of the Smith” for his purposes. The paper was translated from Swedish and proved to be a treasury of mathematical problems on finding volume, identifying different forms, and the like. Sigov concludes with the discussion of the work of one mathematical circle in St. Petersburg. He claims that in junior class circles, content should differ from senior class circles. For the junior age, he recommends all kind of puzzles, jokes (one example: Which mathematical sign should be written between 2 and 3 to receive a number between 2 and 3?), geometrical models, and so on. Importantly, he recommends having students themselves to be in charge of the sessions: “One student after studying the issue using the book later offers the problem to his fellow students” (p. 17).

The theme of problems for fun and entertainment is explored further in an article by I. Chistiakov (1926) from Moscow. This paper provides a plan of mathematical entertainment for a school of the first stage (as it was called then, and which corresponds to the current elementary school plus a few subsequent grades). Here, materials dealing with numbers and operations, algebra and geometry are given in conjunction with the literary sources for the work. Chistiakov does not explain when these materials can be used—indeed, they can be used both outside of the classroom and in the classroom.

Educational journals such as *Physics, Chemistry, Mathematics, Technique in the Labor School* (*Fisika, khimiya, matematika, tekhnika v trudovoy shkole*) published in 1928-1930 three papers on extracurricular activities in mathematics, effectively offering (probably not consciously) three approaches to their organization. Popov (1928) expressed his hope that the new curricula (which appeared at this time) would put an end to any deviations from the "healthy line firmly regulated by this curriculum" (p. 57). Then "without any special obstacle," the educational authority would be able to provide a supplemental education to anyone so interested. The content of the sessions was practically the same as discussed above: reports of the circle's leaders, reports of the circle's members, practical work with tools and instruments, preparation of models, a review of literature, and problem posing. It should be noted that this publication (based on a presentation for teachers) ended with the information that these teachers, while appreciating the importance of extracurricular activities, could hardly be organized because of lack of rooms, the amount of schoolwork, the schedule of the school day, and other factors. The author himself identified as a major predicament the lack of a body of literature that would be helpful for organizing the classes in a variety of forms.

A second, smaller piece written by Agrinsky (1929) divulged a secret that one could hardly imagine while reading the other papers: that is, very many existing mathematical circles appeared in reality to be a "hidden supplement to the rigid week schedule and the mathematics circle could be better called the circle of self-assistance because it is the place where slow learners receive some help” (p. 71). (It is possible to suggest that Popov’s sentence above regarding his hope that a firm curriculum would help avoid deviations was based on his understanding of the real practice.)

After criticizing this practice and making note (as was typical of the time) that all work of the circle must be planned locally so that local specifics can be considered (including, of course, the needs of the local industry and agriculture), Agrinsky discusses mathematical problems, which he names puzzles or even paradoxes, but emphasizes the necessity of systematic proofs. One problem typifies his approach: “Prove that every integer with 6 digits obtained by writing the number with 3 digits twice is a multiple of 7.” After discussing the solution of this problem, the author presents a few more problems for the circle.

The third paper by Nesterovich (1930) speaks of the circles from a broader perspective and often cites pre-revolutionary and foreign literature. In speaking about the circles, the author repeats the cited works in terms of historical themes as well as independent student work, teachers’ reports, and student presentations. Additionally, Nesterovich introduces the idea of multilevel circles. At the first level, according to him, student presentations should be avoided (except for offering something very simple, say, purely biographical), because students do not yet possess “good” language and other students listening to the presentation will only be confused. The second level circle should employ presentations and independent work, offered according to student interest. One task would be for those interested in drawing, another for those “interested in manual labor,” one more for those “interested in art” (p. 61). Finally, the third level—the level of the independent individual or a very small group—includes the study of more difficult mathematical papers.

It is clear that extracurricular activities in mathematics were the focus of attention. Not only was content of these activities discussed, but so were their forms and methods. It is another story that according to the spirit of the epoch, even mathematical circles designed for those interested in mathematics were overloaded with non-mathematical material. Serious reading and study of new material were expected to happen only at the highest level; while problem solving although was discussed (by the same Agrinsky) and recommended, it was not viewed as a key activity.

After 1931

The first of the Communist party's Central Committee resolutions which reformed education in the country appeared in 1931 (Karp, 2010). Innovations inspired by American progressive pedagogy which dominated the country's post-revolution education were declared leftist mistakes. Many ideas, even some originating from before the revolution and influenced by the international reformist movement, were also rejected. Stalin's goal of industrialization—in other words, construction of the industry for military purposes—required the reconstruction of schools to give students solid knowledge and skills and to prepare future engineers for this emerging industry. The model which the educational authority promptly selected to follow became the pre-revolutionary school. There was, however, one major difference: now, the new Soviet school was open much more broadly than before the Revolution. An important new feature of this emerging system was the highest influence of research mathematicians. Mathematics restored all of its former "rights" as the queen of all school subjects and again became a separate and respected (if not the most respected) subject in the curriculum. Mastering mathematical content was the most important task for mathematics teacher education. Meetings and discussions with mathematics researchers became an important part of the routine for preparing new curricula (Karp, 2010). The development of extracurricular mathematics activities became a critical part of school reorganization. All this reorganization, however, was not done overnight.

Grebencha (1934) poignantly comments that the best school students are often not as good at their college counterparts and explains it as follows:

> The reason for it is that the choice of the profession is often done almost randomly without a seriously thought through goal. It is in the secondary school, responsible for the future students of the colleges and technical institutions that an education should go that will identify and develop the students' ability and interest to this or that discipline of the social, general or special group. (p. 65)

Grebencha continues:

> Identification of the interests of student is a responsibility of every teacher of Soviet school, but even that is not sufficient. . . . There are relatively few people with a visible gift, we are meeting them rarely. Educational environment is a factor more important than the innate abilities. (p. 66)

He later concludes that if the pedagogical work is arranged correctly, students can become interested, their intellectual growth will progress promptly, and they will eventually develop into mathematicians and continue their education successfully and consciously at the university.

It is evident that the paradigm had shifted substantially. Now the discourse dealt not with the development of abilities or serving individual needs. Rather, the goal set by the state should be met: mathematicians were needed and, therefore, schools were assigned to prepare future (or potential) mathematicians. Mathematical circles were an important tool for this goal, and a syllabus for these circles was recommended. The following activities were to be offered in such a syllabus:

- student presentations,
- teacher presentations or lectures,

- organization of small seminars (2-3 sessions) devoted to the study of some mathematical field, and
- discussion of and familiarization with the content of new books on school mathematics.

While the list is not new at all, a comparison of this list with similar lists created earlier indicates that independent work (which was understood as manual labor) or the search for the connections with industry were missing. Such rhetoric had in fact been dropped. Students were now required once again to study using books rather that visiting factories and collective farms.

Grebencha suggested three levels of study (an old familiar idea). The first level contained themes and sessions developing students in general and provoking interest in mathematics; in practice, these themes typically came from history. The second level implied a deeper mathematical analysis of the materials studied. Finally, the third level indicated the study of special parts of mathematics and was intended only for those very interested in mathematics.

Examples were provided in the paper to make these ideas clearer. Among the topics for the second level were similarity or loci (as a source, the famous collection of construction problems by Alexandrov [1934] was listed). The third level included elements of the theory of numbers or theory of equations. The planned sessions were theoretical, although interesting problems were mentioned (albeit mainly when mathematical entertainments were discussed).

In the epoch of centralized school curricula, it was natural that for extracurricular activities not only recommendations but also official syllabi were published (although the note was made that they were just approximate). Programs (1935) described two categories of circles:

- mass circles, the goal of which is the general development of the students' mathematics culture and interest; and

- circles for a limited number of students, the goal of which is to assist the most gifted students to attain a deeper knowledge of different special aspects of mathematics.

It was also mentioned that:

> In the process of studies in the mathematical circle the students would obtain:
>
> a) the knowledge of the parts of mathematics which are not included in the standard school curriculum, but can be explored by the methods of school (elementary) mathematics;
>
> b) the ability to read independently the mathematical literature of different difficulty at different levels of the students' mathematical development;
>
> c) the ability to select, systematize, and summarize mathematical materials;
>
> d) the ability to share mathematical thoughts in the form of reports—in written and oral form (for the circles of the senior grades);
>
> e) the ability to apply mathematical knowledge to practical problems. (p. 3)

The list of recommended literature included a few books from similar lists that had been recommended earlier; however, there were many differences. Practical works were included again, but in a different form: it was recommended that students read about them rather than become their participants. Substantial mention was given to the books of the excellent popularizer Perelman as well as the classic book by Ignatiev (1911-1915). The most important new feature seemed to be the explicit discussion of problem solving utilizing existing collections of problems (pre-revolutionary books often compiled to prepare for examinations). Just a few years earlier, the discussion of school mathematics at circle sessions required special excuses to avoid the charge of creating inequity among students.

This increased attention to problems was a part of the prevailing changes not only in politics, but in education as well. In 1934, in Leningrad (St. Petersburg), the first city mathematical Olympiad in the USSR was arranged, and this effected fundamental change in all mathematical extracurricular work. Fomin (1994) notes that in 1933, the “scientific station for gifted students” was created. This experience (according to Fomin)—that is, the experience of extracurricular activities beyond one single school—proved to be important for the organizers of the Olympiad. Indeed, the experience of extracurricular work unbounded by the borders of one school did exist and was of importance. However, the Olympiad proved to be a brand new form—presentations (lectures, reports) were now replaced with problem solving. It was by no means coincidental that among the Olympiad’s organizers were the country’s leading mathematicians, including B.N. Delone, V.A. Tartakovsky, G.M. Fikhtengolts, and others. The role of problem solving was clear for them; it was not coincidental that the famous collection of problems by Delone and Zhitomirsky (1935) appeared almost at the same time. Each of the problems in this collection, according to the authors’ plan, served not only for the development of skill (albeit the most important one), but also called for independent geometrical investigation.

Problem solving was not juxtaposed to other forms of studies and acquainting mathematical knowledge. It is telling that between Olympiad tours (and the Olympiad included three tours, the first of which was open to everyone and the third of which included only winners of previous tours), special lectures were offered to the participants. The winners received mathematical books as prizes. The athletic side of the competition was intentionally limited and winners were not allowed to participate the following year (Fomin, 1994).

In Moscow, the first Olympiad was organized in 1935, and again among its organizers were the most important research mathematicians from the city. As Boltyansky and Yaglom (1965) indicate:

> Yet before the Olympiad a few students of the mathematical department of the University conducted mathematical circles in Moscow schools. After the Olympiad the decision was made to transfer this work to the University and combine it with the lectures which have been offered before at the Mathematical Institute of the Academy of Sciences.

> That is how the school mathematical circle at MSU (Moscow State University) was arranged. (p. 9)

Among the most important organizers of the circle were Moscow mathematicians L.A. Liusternik, L.G. Snirelman, and I.G. Gelfand. These and other outstanding mathematicians offered lectures to the students on a regular basis. Some of these lectures (but not all) were later published. They paved the way to another form of education: booklets in the series "Popular Lectures in Mathematics," which have been published since 1934 for students' independent reading. Boltyansky and Yaglom make an interesting comment that these lectures transformed the understanding of the notion of elementary mathematics. It became clear that some fairly sophisticated topics (such as the proof of the fundamental theorem of algebra or the classification of second-order partial differential equations) could be discussed in a relatively elementary way with school students (albeit gifted and interested students).

Lectures or, more accurately, reports of the students were the most typical form of the circle's work for some time. Boltyansky and Yaglom (1965) note, however, that step by step, it was concluded that this form of work was not the most effective; even if the students understood the material very well, it was not necessarily the case that they would -be able to talk about the material with their fellow students in the best way. Structuring their presentations well and finding the best problems could be challenging for them.

The revolution in teaching circles occurred in 1937-1938 and was led by then-university student David Shklyarskii, who practically put an end to students' presentations. Instead, the circle leader (Shklyarsii) gave a lecture (sometimes over a few consecutive days), followed by problem posing and solving. Sometimes these problems dealt with the material of the lecture, but frequently they offered opportunities for independent mini-investigations. Solutions for the problems were reported and discussed at the circle's meetings. As Boltyansky and Yaglom (1965) point out:

> The benefits of the new system of work were checked by the experiment. In 1937-38 school year Shklyarskii arranged the teaching in his section following the system described above; meantime other sections worked as before, that is mainly offering students' reports. The result was beyond any expectation: at the IV Olympiad (1938) members of Shklyarskii's section got half of all awards (12 out of 24) including all four best awards. . . . These results impressed all section leaders so deeply that since the next year practically all sections followed the new experience. (pp. 25-26)

The Olympiad results were clearly influenced by different factors (Boltyansky and Yaglom themselves mention it), and, of course, it was not possible to judge a teacher only by the Olympiad's results. Still, the new approach was interesting for students, and became increasingly popular, step by step reaching beyond the MSU circle and being introduced to other circles (with all kind of reservations). Kolpakov's (1941) book represents the Leningrad experience of conducting mathematical circles.

One of the important details that cannot be overlooked when analyzing the success of the circles and Olympiads in Moscow and Leningrad is that there were quite a few circles in the country. The miraculously surviving diary of a young man who lived in the 1930s and eventually became an important historian (Man'kov, 2001) describes a life without much place for the circles, for at this time it was difficult simply to survive. The noted St. Petersburg teacher of mathematics Taisiya Kursish, who graduated from the school soon after World War II, recollected:

> There were no circles or anything like that for us then. In general, everything went much more simply than later. I mean all this school education. The lesson was a lesson only. The lessons were pretty boring. (Zakharova, 2015, p. 48)

In reality, the circles still existed in Leningrad at that time. Another famous St. Petersburg teacher of mathematics, Iosif Verebeichik, recalled in his interview (see Karp, 2010b for details of the interview) that he entered the field of mathematics before the war by attending sessions at a scientific station for students. Another individual from St. Petersburg, a research mathematician and a textbook author named Alexey Werner, described his experience during the second half of 1940s and the beginning of the 1950s as follows:

> I was always good at solving problems in arithmetic—these lengthy seven- to eight-step problems. I was good at geometric construction problems based on Kiselev's [author of famous textbooks] textbook in seventh and eighth grades. During the eighth grade, we were visited by people from the mathematics department of Leningrad State University. During that time I began attending a mathematics circle [club]. The circle was taught by Elena N. Sokiryanskaya, a very well-known person for people of my generation. Ilya Bakelman [later a well-known mathematician] used to teach this circle at times. On Sundays, school children were given lectures by Fikhtengolts, Faddeev, and Natanson. I did not plan to pursue mathematics, but to my surprise, I became a winner of the mathematics Olympiad in the eighth grade. As a winner of the Olympiad, I was invited to attend the mathematics circle at the Young Pioneer Palace that was taught by Bakelman and Sokiryanskaya. That is when I began seriously studying mathematics. We covered standard topics in the circle at the Pioneers Palace, that is, topics that are now covered in schools with an advanced course of mathematics. Nowadays, the circles are different, but back then we used to solve problems in geometry from the books of Hadamard (1948, 1952) or of Adler (1940) or covered, for example, circle inversion or the four-color theorem. (Karp, 2014, pp. 184-185)

L. Fedorovich's materials: One case study. Werner's experience, however, was that of a highly gifted student who was an Olympiad winner. "Ordinary" circles for "ordinary" students were in a short supply both before and after the war. Still, mathematical extracurricular activities existed. Important evidence for this is provided by the unique materials of Moscow teacher Fedorovich which have survived in archives and in her paper, published in 1940. This paper was published less than 10 years after the papers discussed in the previous sections, but there is a discernible difference between them. First, they differ in how detailed their recommendations are. Fedorovich (1940) basically provided plans, although short ones, of all of a circle's sessions, themes, and recommended literature. She supported the idea that extracurricular activities should originate in the lesson. As one possible strategy, Fedorovich recommended the following: at a lesson, a teacher can present an interesting problem and ask students to prove some statement (asking students to give differing proofs). As a result, there would be so many volunteers who wish to share their proofs that the teacher can recommend they first write their proofs and then submit their notes. However, this would not resolve the situation; because of the number of possible variants submitted, it would be impossible to listen to all of the students during the class time. The best way to deal with the situation, then, would be to offer students a chance to discuss their solutions after regular hours. This would be a great way to start a circle.

Fedorovich (1940) described several forms of extracurricular activities in addition to the circle. Among them was the mathematical newspaper which can be published by members of a circle or by a specially-created board. Another form was the mathematical evening or event (she suggested performing a scene of an arithmetic lesson from the famous play *The Minor* by Fonvizin). She mentioned mathematical excursions, with a caution, however, that while this form of work can be only rarely used, it can be useful to show mathematical applications in practice. Finally, Fedorovich discussed students' mathematical essays that represent their independent work.

It is interesting that Fedorovich made easy reference to prerevolutionary books or books whose authors were arrested (e.g., popular textbook author Rudolf Gangnus). One may conclude that at that time there was no order to demonstrate political awareness when considering mathematical literature.

Given that Fedorovich was very active in educational and mathematical activities in her district, it is no surprise that many documents describing the work of other teachers survived in her archival collection. That provides an opportunity to analyze not only specially prepared journal publications, but also documents that represent the reality of everyday life—even if it is a glimpse into a slightly improved version of everyday life.

One district school submitted a notebook describing schoolwork in the form of mathematical essays, which were mandatory for every student receiving grades of “4” and “5” (“B” and “A,” respectively). One theme given to eighth graders, for example, was to list properties of isosceles trapezoids. A recommendation was to make use of Rybkin’s standard collection of problems, but the assignment still required some work: it was necessary to collect the properties, organize them in order, generalize, and so on. (One example of a nontrivial property is the following statement: if the side of an isosceles trapezoid equals its midpoint connector, then it is possible to inscribe a circle in this trapezoid [*Dokumenty pedagogicheskogo kabineta uchiteley matematiki*, 1940s, p. 31]).

Another essay was devoted to the regular pentagon. As the teacher offering this assignment wrote herself:

> Students wrote essays following the following plan:
>
> 1. Finding the measure of the internal, exterior and central angle of the pentagon. 2. Symmetry. 3. Properties of diagonals (congruence, dividing each other in golden ratio). 4. Different approaches to inscribing a regular pentagon in the given circle. 5. Construction of a regular pentagon with a given side. (p. 34)

During the 1940s, schools were required to develop the spirit of patriotism. Here is the teacher’s commentary on this:

> It is desirable that the names of the great Russian mathematicians would be present in these essays, that students when comparing contributions made by different mathematicians in the treasury of the world science would see all specific features of the Russian geniuses, including the fact that in many parts of mathematics (as well as in other sciences) our Russian scholars were leaders and pioneers. It goes without saying that when learning about it, students will feel a special love and respect for the giants of Russian mathematics and for all our mathematical scholarship, which always has been advanced even in the conditions of the reactionary tsarist regime. (p. 36)

The essays were not only read by the teacher, but also reported to the larger audience. One teacher commented to this effect:

> This year in response to the suggestion of our school Komsomol [Young Communist Organization] committee the system of lectures was organized at school. All teachers and all school subjects contributed and took part in the organization. That was a great way to familiarize many students with these mathematical essays. (p. 39)

The extracurricular activities employed many forms typical for work with *young pioneers* such as assemblies. A mathematical assembly in the seventh grade was described in another notebook submitted by another school. The theme of the assembly was “applications of mathematics in life.” The notebook contains a detailed plan that indicates responsibility for each stage of the event and the timing of each step, as follows:

1. Parade (5 min.)
2. Introductory note on the application of mathematics to the evaluation of distances (Shekhter, 15 min.)
3. Problem solving with the help of visual aid (Lukashov, 7 min.)
4. Solving puzzles (Minshtein and Gil’shtein, 15 min.) (p. 43)

Two visual aids and a newsletter were also prepared for this assembly. Moreover, the notebook contained a discussion of this event (p. 48), in which it was concluded that the assembly was well organized, although a bit noisy at the beginning. At the end of the note, Fedorovich provided her own review:

> The material for the assembly for the day of Soviet Army is well prepared. It includes a diverse set of problems on the computation and on labyrinths. It is possible to assume that the event was helpful for the students. (p. 48)

Obviously, it is possible to assume that the materials which survived in the archive were random; clearly, there were more notebooks that did not survive. Still, even the surviving materials provide enough information to draw valid conclusions. First, it is clear that teachers were under fairly substantial pressure to arrange extracurricular activities; these notebooks were not likely submitted voluntarily. Second, systematic work in circles with systematic problem solving was not frequent and was hard to arrange both for teachers and students; teachers were often looking for other forms and other activities.

The development of the system in the 1950s. Serebrovskaya's (1948) book is evidence of a further understanding of the importance of circles. As Serebrovskaya wrote:

> In the entire system of all extracurricular activities and events the part of the mathematical cycle is very small, mathematical circles are rare and represent the initiative of few teachers. We cannot say that they represent a systematic and well planned work. (p. 4)

She called for improvement of this situation, explaining that circles were important for all students. To confirm this statement, she cited a letter which she claimed she received from a student from Krasnoyarsk. This girl expressed her disappointment at having received grades of "3" in mathematics because her grandfather was not sufficiently strict with her and permitted such results. The girl reported that she was interested in Russian literature and wanted to become a poet; however, she believed that "mathematics is not foreign for the poet either" (p. 5) and, therefore, wanted to attend a mathematical circle.

The circle discussed here was not a usual one. Serebrovskaya arranged a mathematical circle using radio—quite a new form and approach. She advertised other forms as well: traditional mathematical circles, mathematical events, and all kinds of mathematical competitions. In her book, she offered many materials for all these forms of activities. At first, her book was published in Irkutsk but then was published again in Moscow in a somewhat revised form (Serebrovskaya, 1954).

The importance of extracurricular activities was always emphasized in official documents:

> Extracurricular work provides an opportunity to organize solving of interesting and challenging problems which develop creativity and mathematical thinking of students; it also permits students to study the elements of the history of mathematics and to familiarize themselves with the lives and work of the famous mathematicians, in

> particular, Russian, as well as to meet with some elements of entertaining mathematics (Nikitin, Gibsh, & Fetisov, 1952, p. 15)

If mathematics lessons were relatively free from ideological pressure (Karp, 2007), it was expected that the situation would be corrected by work beyond the regular classroom. Here, many possibilities could arise to discuss the socialist construction and, most importantly, the leadership of Russian mathematics. The Ministry of Education provided guidance for this, which was repeated at the regional level; hence, booklets and books were published regionally as well. Later, they could be republished centrally (as happened with Serebrovskaya's book).

The following are examples of such publications: Lin'kov (1953, 1954), Pilitsova and Byzov (1955), Solontsova (1955), Stratilatov (1955), and Vakhtin et al. (1952). Practically all have a similar structure: first, the importance of extracurricular activities is emphasized, then a variety of forms of these activities is discussed, and finally the materials for this work are provided. Lin'kov (1954), for instance, speaks of circles, mathematical newsletters, mathematical events, show windows, statements of great mathematicians, and finally the mathematical Olympiads. Sometimes, another aspect is added, such as mathematical excursions (Stratilatov, 1955). Other times, some material is given without direct recommendations for how to use it. In the book by Vakhtin et al. (1952), the biography of N.I. Lobachevsky is given and hyperbolic geometry is discussed without any pedagogical recommendations.

The picture of teaching in the cited books represents—albeit in a somewhat improved version—what was happening in the schools. Unfortunately, no statistical data are available to show how widely each form of activity was used. In addition, it is important to mention that after the war, more and more specialized collections of problems appeared to prepare for the Olympiads. The Instructive Letter of the Ministry of Education of the Russian Federation (Glagoleva, 1956) indicated:

> The major goal of the mathematical Olympiad is to increase the students' level of knowledge and to broader their mathematical horizon. That is why the teacher must be very accurate and attentive when selecting problems, each of them should be assessed as to how valuable it is for the educational purposes and how diverse is the set of the skills and abilities required for it solution. . . . The Olympiad should mainly offer tasks which will help students to learn more, to understand the curriculum more deeply, and also to develop special qualifications and skills: such as the development of the logic and the technique of computation, and so on. (p. 8)

The Instructive Letter added:

> The deficiency of each Olympiad is that fewer prepared students are admitted to the next rounds. That is, those are lost for whom the events promoting interest in mathematics would be particularly valuable and who should be brought to the mathematical activities. That is why according to the pedagogical principles the work with those falling behind should not be stopped after the Olympiad. On the contrary, they should be invited to do extra work and solve additional tasks in order to prepare for the next Olympiad. (pp. 3-4)

School Olympiads proved to be fairly difficult to arrange, and teachers' need for additional assistance in organizing and conducting Olympiads soon became apparent . It was recommended that these school Olympiads be conducted in each school at least in order to select participants for the next district tour, but in reality many teachers nominated participants without holding an Olympiad, by using the usual school results. The materials for school Olympiads were prepared and published accordingly. Also, local mathematics and education centers prepared their own materials to help teachers.

Simultaneously, more advanced materials were prepared and disseminated (for example, the set of publications by Young Pioneer Palace, 1953. 1954a, 1954b). These problems were intended to be helpful in the preparation of students for district and other rounds of the Olympiads. In this way, special Olympiad literature for successful preparation began to be developed.

Mathematics Extracurricular Activities from 1956 through 1991

The period of 1956 to 1991 was not at all homogeneous from either political or educational perspectives. This era started with Khrushchev's "thaw" and social rise, which, after the defeat of the Prague Spring in 1968, turned into the so-called Brezhnev's era of stagnation, which was followed by Gorbachev's Perestroika and a new era of hope. Education in general, and mathematical education in particular, were influenced both by state policy and by feelings in the society. This influence was felt in extracurricular activities as well. At the same time, some processes that originated in the previous era continued to develop as well. For the most effective review, the following analysis will not be arranged purely chronologically, but rather according to the directions of changes that occurred.

Professionalization of extracurricular activities. The word *professionalization* seems to be most suitable for characterizing the process that was developing in the Olympiad movement. Olympiads emerged in 1934 as a means to attract students to mathematics, but step by step they transformed into a new form of competition. One important impetus for this transformation was the emergence of the All-Russia Olympiad in 1961 and of the All-Soviet Union Olympiad in 1967. Since 1959, when international Olympiads emerged, participation and victory in them became a matter of prestige for the Soviet Union. Fomin (1994) noted that "In Leningrad the style of Olympiad mathematics changed to its opposite during 3-4 years from 1958 through 1962" (p. 9). He continued:

> Too strong attention to the athletic side of the matter, which was encouraged both in schools and in circles, gradually resulted in Leningrad becoming a cradle of one more unique phenomenon—mathematical athletic professionalism. (p. 10)

The creation of specialized mathematical schools (Karp, 2011; Marushina, 2015) and large networks of circles (Youth mathematical schoolFomin, 1994) contributed to this change.

Schools and circles competed with each other, and towns competed as well. Of course, countries competed as well, and the USSR did not want to lose. As a result, the number of awards and medals as well as the number of representatives at the high-level Olympiad became very important for many school, town, and regional circles. Special coaches emerged who prepared participants professionally for the competitions. Training sessions typical for athletic competitions were regularly employed in preparing for mathematical competitions.

This was a controversial process. Olympiad professionals often came to the circles as teachers and were able to work on a very high level with students on problem solving. They could impose new and excellent problems on seemingly old and routine material. Highly professional coaches appeared bringing with them significant work experience. The following extract from an interview with a leader of the Leningrad and St. Petersburg circles demonstrated this:

> Yet in 1974 I became a secretary of the Jury of the city's mathematical Olympiads. I started teaching circles while I was yet a student of the 10th grade myself and when in 1975 (I was then 18 years old) I was officially accepted to this job, I had already two years of experience under my belt—and I taught circles in different systems including The Palace of Young Pioneers, and in the Youth Mathematical School at the University. I also had a lot of experience in working on the jury. (Rukshin, 2015, p. 124)

It should be noted that this leader has been working in the Olympiad as a coach up to the present.

At the same time, Fomin (1994) noted:

> Given that the motivation for study in mathematics in the circle was mainly based on achieving Olympiad awards, those not that prompt in answering questions or less proficient in solving "awkward" problems offered at Olympiad could be lost and even disappointed in mathematics. (p. 11)

It is worth adding that the habit of solving "quick" problems—that is, problems that can be solved in an hour or two—can be an obstacle in future mathematical research, given that problems in research mathematics sometimes require years to be solved (Karp, 2007).

The problem of paying too much attention to the athletic side of the Olympiads was not only a Leningrad problem. Other towns did not want to fall behind Leningrad with its fantastic results and so copied the athletic system. To oppose this process, new forms of activities appeared such as special summer schools that were devoted to difficult but not Olympiad problems (Sossinsky, 2010).

It should be remembered that in these years, much literature was published for those interested in mathematics, such as booklets in the series "Popular Lectures in Mathematics" and volumes in the "Library of the Mathematical Circle" (see Chapter II) or the more recent "Kvant Library." This literature could be used in extracurricular activities and, to some extent, was of a different style than that of the competitive Olympiads.

Extracurricular activities by correspondence. Following the success of specialized mathematics schools, a new school—by correspondence—was created in 1964. Students admitted to the school received (by mail) the literature and assignments, which they would then submit later, also by mail. The founder of the school, I.M. Gelfand, recollected how he arrived at the idea of creating this school (http://math-vzms.org/50-let-vzmsh):

> The starting point was my conversation in 1963 with my friend Ivan Georgievich Petrovsky, the rector of MSU. Ivan Georgievich attempted to convince me to join Andrey Nikolaevich Kolmogorov who was involved then in the organization of the boarding school at the MSU for students from outside of Moscow. The idea to assist able and interested in mathematics children from all parts of our country who live often in places where no professional help could be arranged was very important for me. Still, after some thoughts I declined to work for this boarding school, but instead offered Ivan Georgievich to organize with his help a correspondence-based mathematical school by correspondence. This idea is very important for me because I lived myself in the years when I emerged as a mathematician in a provincial place where beyond two or three books and the kind attitude of the teachers I had no support at all. I understand how difficult it is to work in conditions like that and how many really talented people necessary for our country we are losing. I believe that the need in the people able and effective (and not only ambitious) is so big that the boarding school cannot satisfy it alone. Why did not I accept the requests to assist in organization of the boarding school? The reason is that in the boarding school students are placed in an artificial and somewhat

> isolated environment and communicate with a limited and specific group of people. They are far away from the natural and usual style of life. Therefore, the school's responsibility dramatically increases, and it is a responsibility not only for their mathematical preparation, but for their general education and character building in such a way that would not prevent them from finding their own way in life and would not be so rigid in defining their future fate. The school by correspondence in this sense is softer and more natural. Additionally, among those admitted to the boarding school typically are students who found their way (or believe that they did it), and usually these students were the winners of Olympiads. Meantime, I believe that we need to increase the general culture rather than prepare future champions as in sport. In this respect, I am not an athletic coach but rather a doctor at the lesson of physical training.

According to the web site of the correspondence school, 1,442 students were admitted at the beginning and most of them came from small towns or villages. Thousands of students graduated from the school. Given the unique nature of this school, special manuals were written and used, and they became popular even beyond the school. In addition, the activity and success of the school made the correspondence form of teaching very popular; similar correspondence schools affiliated with other universities emerged. Also, Olympiads by correspondence emerged and became popular. (Vasiliev, Gutenmacher, Rabbot, & Toom, 1981).

Extracurricular activities in the mass school. Traditions, approaches, and forms of work that emerged or became popular in the previous period continued to be used in this new time period, and even developed to some extent. New technical options were useful for this development: many books were published and in many copies; and human resources improved, given that many people were involved in extracurricular activities (although probably at the higher than mass level). Many of these people became involved later in the preparation of materials for the mass school. Also, materials for schools with an advanced course of study in mathematics (say, collections of problems) could be used to a degree in the mass school as well.

As in previous years, manuals were published for teachers who conducted extracurricular activities, and often these new manuals were not very different from the old ones. The books of

Kolosov (1958), Podshov (1960), and Lazuk (1968) are examples. They all provide a general description of all popular forms of activities, from the circles to the newsletter. They differ, of course, in recommended materials, although even the materials sometimes differ only in their placement within the sequence of activities and assignments (which is still important). Lazuk structures his materials in an unusual manner: he provides many forms under the umbrella of the circles, but subdivides the work in the circles into sections: a section for difficult problems; a section for preparing models and visual aids; a section for discussions, tournaments, and events, and so on. He also suggests the new term *continuous Olympiad by correspondence*, but the form itself includes giving regular assignments to students and is hardly new.

Shustef (1968) rightly notes that "our school is well supplied by the scientifically popular literature which can serve as a foundation for the extracurricular activities." Accordingly, he believed his task was to assist teachers by providing materials for "artistic" work in mathematics—above all, materials for mathematical events. This form has become very popular, as evidenced by many publications in different parts of the country.

The booklet by Abdurakhmanov (1964) showed that extracurricular work in mathematics in school was sometimes combined with manual labor (as was suggested much earlier, but in a different way). Students in the school in Makhachkala (where the author taught) prepared a device for a mathematical electrical test, where students were required to use a small cable to connect two terminals: one corresponding to a question and another to the correct answer. The correct answer selection was identified when a connected lamp turned on. The booklet by Smyshliaev et al. (1965), prepared in the Republic of Mari, discusses all standard sets of extracurricular activity forms, adding the preparation of a special historic-mathematical calendar, broadcasting school mathematical radio programs, and good old works on measurement (under the title of “Extracurricular Activities in the Summer”).

During these years, an abundance of manuals for mathematical extracurricular activities were published. Among them were manuals written by Balk and Balk (1971) and Gusev, Orlov, and Rozental’ (1984), as previously mentioned. At the same time, it should be noted that the large number of published books is evidence that extracurricular activities in mathematics were considered important and teachers were encouraged to organize them. It does not suggest automatically that the work was indeed so popular in schools.

Elective courses. The Soviet educational system attempted to combine the strongest elements of different approaches and, therefore, tried to encourage mass participation and selection while also educating the elite. Extracurricular work at the highest levels (such as in Moscow and Leningrad circles, where only the winners of Olympiads were admitted) could hardly be conducted in every school of the country. At the same time, the lively performances and newsletters praising Soviet mathematicians were hardly the best form to involve students in real mathematical work.

At the very end of the 1960s (Abramov, 2010), the solution for the existing problem seemed to lie in offering a large number of electives in each school. Basically, electives were mandatory only for those who elected to participate (at the beginning of the school year). Indeed, students could decide which electives to attend; however, flexibility in attending electives was substantially smaller than with circles. That is, one could drop out of a circle at any moment and, in turn, could begin attending a circle at any moment.

Generally speaking, electives were not only offered in senior classes, as evidenced by Nikolskaya’s (1991) book for Grades 7-9. Nevertheless, students were usually more interested in mathematical electives at the end of their school education, when they needed to prepare for entrance examinations and had to learn how to solve challenging problems that were not a part of a regular course. Still, it would be too simple to say that all electives were devoted to solving these type of problems. Nikolskaya's book offers typical "supplementary chapters" for a school course, which covers topics not discussed in regular classes, such as mathematical logic or numeration systems with different bases. One can say that this approach to some extent approximates the old one when students were listening to lectures on supplemental topics. The role of problem solving in these electives, however, was much more important than in any previous time, and the topics selected for the electives were typically closer to the standard school course than was typical for the 1920s and 1930s. Perhaps the highest level of difficulty in elective courses is demonstrated in the books by Sharygin (1989) and Sharygin and Golubev (1991), which offer very challenging tasks from classical school mathematics.

Mathematics Extracurricular Activities after the Collapse of the Soviet Union

The collapse of the Soviet Union substantially changed the lives of its people. The uniform educational space (which basically existed earlier) collapsed as well. Of course, students in Lithuania were always taught in a different way than students in Moscow, even Irkutsk and Leningrad (who belonged to one republic) had differences, but all these differences were relatively minor compared to what was emerging in this new era. Teaching in now independent states soon followed new models, so only Russia (rather than all territories of former Soviet Union) is the focus of this study after 1991.

Many changes occurred in Russia in 1990s: the system became less centralized, and many previously unfeasible things became possible, but at the same time many actions, procedures, and events which had been routine now became difficult to carry out.

The latest changes were most visible in the book market. Books once published by the hundreds of thousands of copies and offered in any part of the country were now published as a few thousand copies at best. For example, 680,000 copies of Galperin and Tolpygo's (1986) book were printed, but only 5,000 copies of a similar book by Fomin (1994) were printed. Another example of changes that negatively influenced extracurricular activities was the mass emigration of mathematicians which started when the Iron Curtain collapsed. Accordingly, the number of people who were instrumental in developing and supporting all extracurricular mathematical activities dramatically decreased.

However, because many activities became easier, many new forms of extracurricular activities emerged. Among them were new mathematical festivals or events which were and are arranged in Moscow for students of many schools (Yaschenko, 2005), or the Olympiad *Kangaroo*, now the center of the organization which emerged in St. Petersburg (Bashmakov, 2010; Bratus' et al., 2003).

Old forms of the work nonetheless continue to exist. The book written by Sheynina and Solovieva (2005) provides yet another example of an approach to organizing a mathematical circle. This book includes plans for 30 lessons for a school circle in a "regular" (non-selective) school. The goal of the book is to provide materials for teaching circles under these conditions. Correspondingly, the book includes fewer problems but more tasks in the form of games or theatrical forms and the like. Another example was Kozlova's (2008) book, which is intended for a mass audience but is more mathematically rigorous than Sheynina and Solovieva's book.

Spivak (2009) includes some problems on Olympiad themes, but these problems are easier than those found in Fomin et al.'s (2004) book. Even more importantly, Spivak's book also includes many tasks "for fun," that is, all kinds of puzzles accessible for a mass audience.

The tradition of mathematical evenings and other events in schools continued as well. Falke (2005) cites plans for a few mathematical "events" to be organized in the schools (one, for example, with the title "Mathematics and Beauty"). At these events, all kind of mathematical games can be offered, some of which are described in the book. These forms of work are intended for mass audiences. It is by no means a coincidence that Petrenko (2011) begins her description with her experience of discussing the increase in the number of juvenile criminals. She believes that engaging students in meaningful mathematical extracurricular activities can prevent any further increase of delinquency among youth. Petrenko views festivals and events as a major means of such reform. Pestereva et al. (2009) lists among their recommendations practically all the forms of activities mentioned earlier; the very first article in this collection is titled "Mathematical Extracurricular Activities in School: Traditions and the Current State," which communicates that the tradition is still alive. But it is another story to determine just how many children are involved in these existing forms of work.

Most importantly, the technological revolution has made possible brand new approaches and forms of extracurricular activities. For example, Sushentsova (2012) describes the use of e-mail correspondence to organize extracurricular activities in Grades 5-6, while being guided by the teacher and upper-grade students who exchange e-mails on various mathematical topics.

In practice, forms that are substantially less difficult to arrange have become popular. Among them are contests and competitions organized via the Internet, some online courses, and forms blending online activities with traditional forms. One example of such an approach can be found on the site http://metaschool.ru/ which offers online competitions and so-called mathematical vacations that are akin to mathematical camps during school vacations. Probably, the most important contribution of technology, however, is in how it makes available to all school students interested in mathematics a vast amount of literature in any place that has Internet access. These changes, for the time being at least, allow this brief history of extracurricular activities to end on an optimistic note.

Chapter V

FORMS OF EXTRACURRICULAR WORK IN MATHEMATICS

This study attempts to address the following research questions:

- What topics are typically covered and what problems are typically offered to students for mathematical extracurricular activities in Russia?
- What are the most important forms of organization utilized in the system of Russian mathematical extracurricular activities?

In the next sections, the organizational forms of extracurricular activities that existed and continue to exist in Russia will be described, with a focus on the most prominent and popular. Specifically, the mathematical content of these activities are considered. To this end, materials from the previously published work of Marushina and Pratusevich (2011) are often cited. Selective and non-selective forms of activities are distinguished, and new forms that became popular over the last decades are described.

Selective Forms of Working with Students

The selective forms of Russian (Soviet) extracurricular activities are widely known throughout the world. Given that the Russian experience of working with highly gifted students is internationally recognized, the description below will be relatively brief; more detailed information about many of the issues raised below be found in Fomin et al. (1998).

Fomin (1994) described the forms of mathematical work in selective circles. These circles are organized either in specialized mathematics schools (Marushina, 2015) or more frequently in centers outside particular schools in large cities. The Olympiads play a large role in the process of selection and a few words should be said about them here.

On the Olympiads

Today the Olympiads are organized in a few rounds. Typically, the first round is the so-called *district-level round*, although officially it is called the second round—meaning that the first round is the school Olympiad. The school Olympiad is supposed to be organized to select participants for the district round. In reality, teachers often select future participants themselves without any additional competition, although, again, they are encouraged to do so differently, and special published manuals and consultations are available to assist them. Good student results at the Olympiads are extremely valuable for teachers; for instance, they are taken into consideration when teachers apply for a promotion or raise in their salary.

The district-level rounds of the Olympiads include problems which, even though they are not especially difficult in general, nonetheless differ substantially from the problems ordinarily solved in the classroom. For example, consider the following problem from a district-level round for sixth graders (Olympiad problems are discussed further in the next chapter):

> Each one of three players writes down 100 words, after which their lists are compared. If the same word appears on at least two lists, then it is crossed out from all lists. Is it possible that, by the end, the first player's list will have 54 words left, the second player's 75 words, and the third player's 80 words? (Berlov et al., 1998, p. 15)

The solution of the problem is based on a simple line of reasoning. If the described outcome were possible, then the first player would have 46 words crossed out, while the second player and third player would have 25 and 20 words crossed out, respectively. But 20+25 is less than 46.

Therefore, not all of the 46 words crossed out on the first player's list could have been on the other players' lists. Although no special prior knowledge is required to solve such a problem, those who have had experience with solving problems that are not "school-style problems" have found themselves in a better position at the Olympiads.

The winners of the district-level Olympiads are invited to the city-level round. Then regional and All-Russia Olympiads follow (formerly, the last step was the All-Union Olympiad). Typically, the winners and awardees of the city-level rounds of the Olympiads are invited to the selective circles (however, sometimes, circles are formed for children who do not yet have Olympiad experience, e.g., fifth graders).

On Mathematics Circles

Mathematics circles are, of course, the most popular form of extracurricular work. In large cities (Moscow, St. Petersburg, Yaroslavl, Krasnodar, Kirov, Chelyabinsk, Irkutsk, Omsk, and others), mathematics circles occur at a city-wide or region-wide level. Study in such circles is intended to take place over several years. Such circles are attended by children from many schools who are, as a rule, ready to spend much time not only on solving problems and studying theory, but also on commuting to the locations where their mathematics circles meet, which can consume a considerable amount of time. In cities other than Moscow and St. Petersburg, work in mathematics circles usually revolves around a very small number (sometimes one or two) of qualified teachers, who engage in the painstaking work of educating gifted children over a period of many years. The graduates of such circles, however, are often among the winners of the International Mathematics Olympiads.

Mathematics circles of the high level, to some extent, compete with one another. Still, it is not a mistake to say that their curricula closely resemble each other. The first years of study are typically devoted to the following themes (not necessarily in this order), as listed in the book by Fomin et al. (1996):

- Elements of number theory. Divisibility. Systems of numeration
- Even and odd numbers
- Introduction to combinatorics
- Graphs
- The Pigeonhole principle
- Invariants
- Inequalities
- Geometrical inequalities

Step by step, the curricula are enriched, offering more and more challenging problems and discussing more and more sophisticated theoretical material. According to Maxim Pratusevich, Principal of Lyceum 239 in St. Petersburg in the city center affiliated with the school, students acquire a thorough grounding in geometric transformations (including inversions, affine, and projective transformations), discrete mathematics, groups, rings, fields, calculus, elementary general topology and functional analysis, and combinatorial geometry.

According to Pratusevich, the circles in the center are arranged so that they meet twice a week, after basic school classes end. These meetings may occur in a variety of formats; for example, they may be organized as:

- lectures on theory;
- individual problem solving;
- discussions of solutions to problems with teachers;
- solving problems collectively, in groups;

- analysis of solutions by the instructor;
- interviews and exams on theory;
- seminars;
- student reports, summaries, and independent projects and research; and
- mathematical competitions.

Each session begins by thoroughly hearing each child's solutions to all the problems assigned to him or her at the end of the previous session. Such work requires the participation of a large number of volunteers—usually older schoolchildren or university students who serve as assistants to the teacher of the mathematics circle. After this, the teacher presents the solutions to the problems on the blackboard, with requisite theoretical commentary.

Mathematics Summer Camps

One more component within the structure of multi-year mathematics circles are intensive summer classes, which take place in so-called mathematics summer camps. In the USSR, there were camps for Young Pioneers in the countryside where students could go on their summer vacations. These camps were inexpensive because they were supported by the state. During a single camp session, which usually lasted three weeks or slightly longer, students would be fed, given a place to sleep, and offered an array of recreational and health-improving activities. At a certain point, there developed a tradition of organizing a mathematics summer camp on the grounds of some camp of the sort just described. During the post-Soviet period, many Young Pioneer camps were destroyed, while the ones that survived were reorganized. However, the tradition of mathematics summer camps survived.

Ordinarily, teachers of the mathematics circles, after arranging a place and time for a mathematics summer camp, assiduously invite the participants of their mathematics circles to attend the camp. Life in the camp is sufficiently close to life in a Boy Scout camp, except that the mathematics cohort (if the camp is not entirely mathematics-based) do mathematics while the rest of the children go on field trips, participate in non-mathematical clubs, or simply play or run around the camp. The mathematical cohort might spend about six hours daily doing mathematics.

The program of these summer classes are typically structured in the same way as ordinary mathematics circles, which is to say that students solve and analyze a great number of problems. Sometimes during summer classes, teachers make theoretical presentations on different topics in mathematics. In any case, it is evident that these three weeks of intensive study are of great importance for the mathematical growth of the students who attend these camps.

Conferences

A form of extracurricular work that is in some sense the opposite of the mathematics Olympiads are student conferences. If in the Olympiads the athletic-competitive element is emphasized, then the aim of student conferences is to encourage the students' scientific work and bring it to conclusion. Reports, which at one time were the main form of work in mathematics circles, now reappear but in a different capacity. Ideally, the students report about their own results.

Among the conferences a special mention should be made of the Festivals of Young Mathematicians, held for many years in Batumi thanks to the energy and initiative of a local teacher, Medea Zhgenti, with the support of the editorial board of the magazine *Kvant*. During the late 1970s and 1980s, these events were held in November during school vacations, and they were attended by teams of students from cities of the Soviet Union. The program included many reports by students that were heard by the participants and a jury, whose core was usually composed of members of the *Kvant* editorial board.

With the collapse of the Soviet Union, the festivals in Batumi ended, but other all-Russian or municipal conferences appeared (for example, in St. Petersburg, conferences were held around a number of schools, such as Physical Technical School #566 or the Anichkov Lyceum). At a certain point, this format became popular.

Karp (1992) described the process of report preparation by students of specialized mathematics. Stages in the preparatory process include the preparation of presentations for a class or a mathematics circle based on existing publications (effectively, the retelling of these publications), the assembly of a bibliography on some topic, the preparation of a summary paper consisting of a compilation of several different publications, and so on. Schools—at least schools with advanced courses in mathematics—must work to develop in their students advanced skills. Still, the preparation of a report for a prestigious conference usually requires even more than this: namely, an independent result, however minor. This presupposes individual work with a scientific advisor who poses a problem and guides the student.

As a whole, of course, the popularity of conferences, even at their peak, was always noticeably lower than that of the mathematics Olympiads. A note should be made that some attempts were made to combine these two forms of events. One of the relatively recently emerging Olympiads is the Tournament of the Towns, which includes not only individual competition but also a competition between different towns, so that the best results of the students from each town (computed according to a fairly sophisticated system that takes the size of the town into account) are summed. As the website of the Tournament noted: “The particular feature of the Tournament of the Towns is that it emphasizes not the athletic success but rather the deep work on problem solving and in this way develops the features necessary for the researcher.” The authors of the best work from Grades 9-10 are invited to the Summer

Mathematical Conference of the Tournament of the Towns. At the conferences of the Tournament of the Towns, participants are given research problems, that is, problems they have to solve over a comparatively lengthy period such as a week (examples of such problems appear in Berlov et al., 1998). These problems basically represent serious research questions.

Extracurricular Work for All

Russian extracurricular mathematical work with “ordinary” students is less known internationally. This work is valuable for maintaining the high reputation of mathematical studies and expresses the broad understanding of its importance. As mentioned above, extracurricular activities were usually considered for their connections with classroom work. Indeed, class work usually includes (or should include) problems of different levels of difficulty that might pique students’ interest. Special supplementary sections in textbooks, which contain optional materials addressed only to those who want to learn, serve the same purpose. A teacher may suggest that students prepare a presentation on some topic, thus giving them an opportunity to become acquainted with additional literature outside of class. All this can smoothly be transferred to the organization of the circle or any other form of extracurricular activities.

Mathematical Wall Newspapers

Lin’kov (1954) explains that these newspapers can be published either by a mathematical circle or by a group of interested students who form an editorial board. As an example, he describes newsletters published in schools of his native town Kursk. These newsletters included the following materials:

- elements of the history of mathematics;
- problems corresponding to the regular curriculum but more challenging;

- puzzles;
- problems-jokes, mathematical sophisms, etc.; and
- correspondence with readers.

These or similar themes were offered by other authors describing the school mathematical newspapers. The content of such a newspaper may vary, but it is clear that it must, on the one hand, attract attention and, on the other hand, be sufficiently easy and quick to read—standing in front of a newspaper for hours is hardly feasible.

One should logically not expect that reading a wall newspaper in itself will steer students toward doing mathematics on their own. The goal here is different: it is to attract students' attention and perhaps to inform them about other extracurricular activities being offered. At the same time, if a wall newspaper is published regularly, then it usually acquires an editorial board: certain students who systematically choose materials for it and gain a considerably deeper acquaintance with these materials in the process. This, of course, concerns only a small group of students.

Mathematical Theatrical Evenings and Oral Mathematics Journals

The activities discussed in this section can go by different names, but all of them involve asking students to participate (on the stage or as members of an audience) in a theatrical presentation. Most often, such forms of extracurricular work are used with students in Grades 5-7. The purpose is not so much to teach the students mathematics, nor is it necessarily even to interest students in mathematics as much as it is to demonstrate the "human face" of mathematics.

The script of a mathematical theatrical evening may include the following sections (Falke, 2005):

> Presentations about mathematics delivered from the point of view of other school subjects (mathematics and Russian literature, mathematics and physics, etc.);

A parade of the “components of mathematical beauty” (students who represent symmetry, proportion, periodicity, etc., tell about these concepts, offering examples);

A reading of poems about mathematics;

A story about some great mathematician;

Scenes with mathematical content, performed by the students;

Mathematical questions for the audience; and so on.

Naturally, for such a theatrical evening to be a success, it is necessary to write a good script, do a good amount of rehearsing, possibly prepare costumes, and so on. While none of these activities are usually considered mathematical, it may be expected that the teacher who has undertaken to supervise them will endow the students with a positive attitude toward studying mathematics.

Stepanov (1991) describes an “oral journal” for seventh graders in a school, the purpose of which was to publicize a new mathematical elective being offered: “The pages of the journal were given to a ninth-grader (‘Sufficient Conditions for Divisibility’), an eighth-grader (‘How People Counted in Ancient Russia’), a mathematics teacher (‘Symmetry in Mathematics and Around Us’), and to the economist-parent of one of the students” (p. 6). After the conclusion of the journal, the program of the new elective was displayed and students had a chance to sign up for the course.

Actual mathematical activity—problem solving—is usually not a large part of such theatrical evenings. The “questions for the audience,” mentioned above, may be completely elementary: “Can the product of two whole numbers be equal to one of them?” or “Is the difference of two natural numbers always a natural number?” (Falke, 2005, p. 28). However, a theatrical evening may also include a small competition in which students solve more difficult problems.

As an example of such an entertaining and comparatively easy problem, consider a question given at the Mathematics Festival in Moscow, which constitutes a special Olympiad for Grades 6-7:

> A kilogram of beef with bones costs 78 rubles, a kilogram of beef without bones costs 90 rubles, and a kilogram of bones costs 15 rubles. How many grams of bones are there in a kilogram of beef? (Yaschenko, 2005, p. 10)

To solve this problem, it is enough to notice that a whole kilogram of beef costs 75 rubles more than a kilogram of bones, and 12 rubles more than a kilogram of beef with bones. Consequently, the share of bones in a kilogram of beef with bones is $\frac{12}{75} = \frac{4}{25}$. From this, it is clear that a kilogram of beef with bones contains 160 grams of bones.

Mathematical Tournaments

By contrast with mathematical theatrical evenings, mathematical tournaments are entirely devoted to competitive activities, which may be conducted, for example, in a class and consist of answering engaging questions. The questions for such contests may be prepared by the teacher or by the students themselves. A mathematical tournament may involve the participation of the whole class, for example, divided into two or three teams. Verzilova (2007) offers a detailed description of such an event for sixth graders, which is reproduced in abridged form below:

> The program for the event is put on display one week before the event takes place. The teams are given homework assignments (see below). A panel of judges consisting of students from higher grades is prepared. After an opening statement from the master of ceremonies, the competitions begin. They include the following:
>
> *"Auction."*
>
> A set of triangles is put up for auction. The teams take turns stating facts about the topic: "Angles" (the homework assignment included a review of this topic). The last team that can state a fact about angles wins the set of triangles.
>
> *"Experiments with a sheet of paper."*

The teams have several sheets of paper, some square and some irregularly shaped. They are given the following assignments:

1. Fold a sheet of paper to obtain a right angle.
2. Fold a sheet of paper to obtain a 45º and a 135º angle.
3. Fold a sheet of paper to obtain a rectangle.
4. Take a square, fold it along its diagonals, and cut it along the lines of the folds. Using the obtained shapes, assemble: (a) two squares; (b) a rectangle; (c) a triangle; (d) a quadrilateral that is not a rectangle; (e) a hexagon.

"Eye test."

Several different angles, made of transparent colored film, are projected onto a screen. The members of all of the teams are asked to estimate their degree measures and write them down on a sheet of paper. Then, using a transparent protractor, the angles are measured and all of the participants write down the correct results next to their guesses. The sheets of paper are then submitted to the judges to determine who had the most right answers.

"Scientific fairy tales."

Each team is asked to read two fairy tales, which they have composed in advance as part of their homework assignment. The remaining fairy tales are given to the judges to determine the winners of the homework competition. An example of a fairy tale composed for such an event:

Adjacent Angles.

Once upon a time, two angles lived in the same house. They did not look like one another, because one was obtuse and the other acute. Their names were: angle AOB and angle COB. It was impossible to separate them, since they had one side in common, and their other two sides formed a straight line. The angle brothers got along very well together, never leaving each other's side. Most of all, they wanted to invent a name for their house. They thought for a long time and finally decided to name their house after themselves: "Adjacent Angles."

Several other contests follow. The mathematics festival concludes with the judges determining the winners and handing out awards.

Written problem-solving contests

Optional problem-solving contests are conducted in a class (or a school). Of course, not all students take part in such contests (let alone successfully solve all problems), but all students in a class (or a school) are invited to participate in them, and that is why this activity is discussed in this section. Such contests are useful both in themselves and as a means of drawing students into a mathematics circle (where, for example, they will be told the solutions). Contest problems may be given in wall newspapers. They may be given one or two at a time, for example, as weekly assignments. Whatever the case, they are usually given for a sufficiently long period of time and thus presuppose that the participants have attained a certain degree of maturity and responsibility. It must be pointed out, too, that students are almost always unaccustomed to turning in work in which not all problems have been solved (and usually even the winners do not solve all of the problems). Consequently, it is very important to explain to potential participants that they are in no way expected to solve all of the problems.

In general, the psychological aspects of such contests usually require a fair amount of attention. If a contest turns out to be too easy, then the stronger students will not want to solve and hand in the problems; if it turns out to be too difficult, then, on the contrary, no one except a very small number of students will decide to participate in it. Consequently, a certain balance is necessary. Likewise, a balance is necessary between comparatively traditional, "school-style," problems and problems with interesting but unfamiliar formulations.

The following problems were used in a contest for seventh graders (Karp, 1992, pp. 10-11):

1. Solve the equation $|x-1|+|x+2|=4$. (This is a typical, "difficult" school-style problem. The students have already analyzed absolute value problems, but even a single absolute value in seventh grade made a problem difficult, while this problem contains two of them. On the other hand, there is nothing particularly unexpected here: carefully following the algorithm for removing the absolute value sign, for example, will lead to the right solution.)

2. Two squares, with sides 12 cm and 15 cm, overlap. Removing the common part from each of the squares, we obtain two regions. What is the difference of their areas equal to? (In this case, for a person with a certain mathematical background, everything is very straightforward: regardless of the area of overlap of the two squares, the difference of the areas of the obtained regions is equal to the difference of the areas of the squares. But far from all students are capable of justifying this argument clearly and correctly.)

3. A scary dragon has 19 heads. A brave knight has invented an instrument that can chop off exactly 12, 14, 21 or 340 heads at once, but after this the dragon grows 33, 1988, 0 or 4 new heads, respectively. Once all of the heads have been chopped off, no new heads will grow. Will the knight be able to slay the dragon? (This is a typical, although not difficult, Olympiad-style problem: the students must notice that the number of heads always changes by a multiple of three, and therefore there is no way to pass from 19—a number not divisible by 3—to 0.)

School Mathematical Olympiads

It has already been mentioned that few schools conduct their own Olympiads (although all are encouraged to do it). Schools with an advanced course of study in mathematics are among them; moreover, they often conduct a few rounds of their own Olympiads. Even the first round of these Olympiads is typically close to the level of district Olympiads, whereas the Olympiads in “ordinary” schools are typically substantially easier. Farkov (2004) offered a few tasks for these Olympiads. Typically, there are five to six problems in the set, and a complete essay-style response showing all steps of the solution is required. Typically, the time allotted for the Olympiad varies from 1 hour (in Grade 5) to 2-3 hours (in Grades 9-11). Problems are typical traditional and challenging algebra, pre-calculus or geometry problems.

The following task is one example:

Graph the following function: $y = \frac{x^2 - x}{x^2 - 1} + \frac{x^2}{x+1}$. (Farkov, 2004, p. 45)

The student is required to conduct some transformation and obtain the linear function. Two points should be excluded from the graph (corresponding to the values of x where the given function is not defined).

School Mathematics Circles and Electives

This section describes school mathematics circles and electives. It is necessary to emphasize once more that the circles in “ordinary” schools rather than in specialized mathematics schools (Vogeli, 2015) are discussed here. The participants in these circles are not the future winners of the All-Russia or International Olympiads, but rather “ordinary” children, and the goal of the circles is not a preparation for the high-level Olympiads or a high-level study of mathematics, but rather the motivation of students for the future study of mathematics.

Officially, the differences between mathematics circles and electives have been (and remain) substantial. Generally speaking, students have the right to choose which electives they wish to attend, but once they make their choice, they are required to attend the elective they have selected; by contrast, participation in mathematics circles remains voluntary at every stage (to be sure, a teacher can, in certain situations, prohibit students who skip mathematics circle meetings too often from attending at all). The wages that teachers receive for teaching mathematics circles and electives differ as well (and it should be noted that teachers have sometimes taught mathematics circles with no compensation at all). Nonetheless, it is not always possible to make a sharp distinction between the programs of mathematics circles and the electives. A topic that has officially been included in the program of electives may become the basis for a mathematics circle. The more adult word “elective” is heard more frequently in the higher grades; in grades 5-6, only the term “mathematics circle” is used.

Mathematics circles in grades 5-6. Sheinina and Solovieva’s (2005) manual provides a rough description of the work of such a mathematics circle. The manual contains material for 30 sessions (1.5 to 2 hours each) and, as the authors remarked in their annotation:

> It was written with the aim of helping the teacher of a school mathematics circle to conduct systematic sessions (at least two per month) [whose purpose is] to interest the students, supplementing educational material with facts about mathematics and mathematicians, to improve students’ mental arithmetic skills, to develop their basic mathematical and logical reasoning skills, to expand their horizons, and above all to awaken their interest in studying one of the basic sciences [namely, mathematics].

As an example, consider one mathematics circle session outlined in the book (No. 14). The session consists of several sections, for which materials are provided. At first, students are given various puzzles, among which may be the following problem:

> Express the number 1000 by linking 13 fives in arithmetic operations (for example, $5\cdot5\cdot5\cdot5+5\cdot5\cdot5\cdot5+5\cdot5\cdot5\cdot5+5\cdot5\cdot5\cdot5$).

Next come several "fun questions":

- Five apples must be divided among five children so that one apple remains in the basket.
- Two fathers and two sons shot three rabbits, one each. How is this possible?
- How many eggs can be eaten on an empty stomach? and so on.

(The answers are, respectively: one child must be given the basket with one of the apples inside it; the rabbits were shot by a grandfather, a father, and son; and only one egg can be eaten on an empty stomach).

Next, the students are given a brief biography of Newton, followed by a section called "Solving Olympiad Problems." Here, students are asked to use trial and error to find the solutions for the equation $2y = y^2$, to solve a rather lengthy word problem, and to say whether a boy has 7 identical coins if he has a total of 25 coins in denominations of 1, 5, 10, and 50 kopeks. The session concludes with a poetry page: the students read a poem about the Pythagorean theorem, and so on.

The methodology of conducting a mathematics circle session is not discussed in the manual, but it may be assumed that, for example, the biographical vignette is presented by the teacher or by a specially prepared student. The poetry page is likely approached in a similar fashion.

The plan of the session described above is traditional. Lin’kov’s (1954) book, written half a century earlier, offers similar plans. For example, the ninth session of the Grade 5 circle first offers a talk about “the academician I.M. Vinogradov, the strongest mathematician in the world,” then one about Eratosthenes’ sieve, and finally a game of solving a Russian folk problem “Birds and Sticks” (p. 10).

The examples given above show that the work of a mathematics circle can hardly be characterized as intensively mathematical: rather, the work is focused on the students’ general development. Nonetheless, mathematics circles play a clear role in instilling in students a positive attitude toward problem solving and studying mathematics in general.

Mathematics circles and electives in grades 7-9. In working with students from grades 7-9, less attention is naturally devoted to the “entertaining” side of the subject than it would be in working with students from grades 5-6. The program of study becomes more systematic for the upper grades. Nikolskaya’s (1991) manual, published in Soviet times, recommends the following program of study for elective courses in these grades:

Grade 7

- Number Systems
- Prime and composite numbers
- Geometric constructions
- Remarkable points in a triangle

Grade 8

- Number sets
- The coordinate method
- Elementary mathematical logic

- Geometric transformations of the plane

Grade 9

- Functions and graphs
- Equations, inequalities, systems of equations and inequalities
- Remarkable theorems and facts of geometry
- The logical structure of geometry

In other words, such a program involves expanded study of the existing school program (indeed, since the collapse of the Soviet Union, with schools acquiring greater opportunities, such a program or one similar to it has sometimes simply been added to the ordinary school curriculum, with the classes that study this expanded curriculum being labeled as classes with an advanced course in mathematics).

The manual by Gusev et al. (1984), published even earlier, suggests a number of topics for extracurricular work in grades 7-9 (or 6-8 in the system that existed at the time), which largely resembles the topics found in mathematics Olympiads. Among them, for example, are such sections as “Graphs,” “The Arithmetic of Remainders,” “How to Play in Order Not to Lose,” “The Pigeonhole Principle,” and so on. For each topic (which usually occupied several class sessions), the manual offers problem sets and provides specific methodological recommendations, suggesting various general theoretical facts that the teacher can convey to the students in one way or another, or describing various kinds of activities that the teacher might organize.

In fact, during those years as well as in later years, students in school electives and mathematics circle sessions usually solved problems of a heightened level of difficulty. For the most part, these problems were based on materials from the ordinary school curriculum, but they could also include problems that drew on traditional Olympiad-style topics. For example, problems involving absolute value or problems that required students to construct nonstandard graphs (e.g., construct the graph of the equation: $y+|y|=x$ [Kostrikina, 1991, p. 46]) have always been very popular. The same is true of problems requiring the solving of equations and inequalities, as well as word problems based on equations and inequalities. Also represented are identity transformations, problems on progressions, and trigonometry. Kostrikina's problem book, cited above, contains problems of a heightened level of difficulty in practically all sections of the course in algebra. Consider several more examples of problems from this text:

- Find a two-digit number that is four times greater than the sum of its digits. (p. 43)
- What is greater, $\frac{10^{10}+1}{10^{11}+1}$ or $\frac{10^{11}+1}{10^{12}+1}$? (p. 48)
- Simplify the expression $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}$, if $1 \le x \le 2$. (p. 102)
- For what value of *a* is the sum of the squares of the roots of the equation $x^2+(a-1)x-2a=0$ equal to 9? (p. 108)
- Prove that the greatest value of the expression $\sin x+\sqrt{2}\cos x$ is equal to $\sqrt{3}$. (p. 180).

The first two of these problems are recommended for grade 7, the next two for grade 8, and the last one for grade 9. As can be seen, these and similar problems place rather high demands on students' technical skills, but the reasoning skills required to solve them are also very high (of course, students are also given simpler problems to solve in mathematics circles and electives—the examples above were chosen to illustrate the types of problems offered).

There is a considerable amount of material in geometry for school extracurricular work. The curriculum for grades 7-9 contains a sufficiently complete and deductive exposition of

Euclidean plane geometry; this material may be used as a foundation for posing problems that are varied in character. Indeed, school textbooks themselves usually provide considerably more material than can be studied and solved in class. Among the supplementary manuals, a note should be made about the popular and frequently reprinted problem book by Ziv (1995), intended for use in ordinary classes, but containing more difficult problems recommended for mathematics circles. Again, since lack of space prevents a more detailed description of these problems, only one example will be presented:

> A point D is selected inside a triangle ABC. Given that $m\angle BCD + m\angle BAD > m\angle DAC$, prove that $AC > DC$. (Ziv, 1995, p. 59)

The solution of this problem, which is assigned to seventh graders, is based on the fact that the longest side of a triangle lies opposite the largest angle and on the properties of a triangle's exterior angle. However, to arrive at this solution, students must possess a certain perspicacity and, above all, a comparatively high level of reasoning skills. Solving geometric problems as part of extracurricular work (and usually in classes as well) practically always involves carrying out proofs of one kind or another.

Evstafieva and Karp's (2006) manual provides an idea of what kind of typical Olympiad-style materials might be studied in mathematics circles. This collection of problems, intended mainly for working with ordinary seventh graders in ordinary classes, contains a section entitled "Materials for a Mathematics Circle." This section has five parts:

- Divisibility and remainders;
- Equations;
- The Pigeonhole Principle;
- Invariants; and

- Graphs.

As can be seen, the topics are traditional for mathematics circles of even higher levels (Fomin et al., 1996). But here the assignments are limited to relatively easy problems, the number of which, however, is relatively large and organized in such a way that, after analyzing one problem, students can solve several others in an almost analogous fashion. For example, the following three problems appear in a row:

- The numbers 1, 2, 3, 4 … 2005 are written on the blackboard. During each turn, a player can erase any two numbers x and y and write down a new number x+y in their place. In the end, one number is left on the board. Can this number be 12957?
- The numbers 1, 2, 3, 4 . . . 2005 are written on the blackboard. During each turn, a player can erase any two numbers x and y and write down a new number xy in their place. In the end, one number is left on the board. Can this number be 18976?
- The numbers 1, 2, 3, 4 . . . 2005 are written on the blackboard. During each turn, a player can erase any three numbers x, y, and z, and write down two new numbers $\frac{2x+y-z}{3}$ and $\frac{x+2y+4z}{3}$ in their place. In the end, two numbers are left on the board. Can these numbers be 12051 and 13566?
(p. 150)

A problem of the same type as the first of these problems (although slightly more difficult) also appears in the aforementioned manual by Fomin et al. (1996). The solution to the first problem above is altogether simple: the sum of the numbers on the board does not change after the given operation, and consequently the number left on the board at the end must be equal to the sum of all the numbers that were on the board at the beginning, which is obviously not the case if the last number is 12957 (note that the problem is posed in such a way that this answer is obvious in the full sense of the word—it is not necessary to find this sum). But in the problem book that is aimed at a more selective audience, the very next “similar” problem is far more difficult, whereas in the case above, it is relatively easy for students to determine what remains invariant in the subsequent problems; they might be asked to invent an analogous problem on their own, and so on. In other words, the goal is not so much to solve increasingly difficult problems by using a strategy that has been learned as it is to become familiar with this strategy itself—in this case, with the concept of invariants.

Thus, the topics studied in mathematics circles are often mixed, including a fair number of Olympiad-style problems and typical difficult school-style problems.

Mathematics circles and electives in grades 10-11. Despite lacking any firm statistical evidence, it is still possible to argue that mathematics circles and electives in the higher grades of ordinary schools are devoted mainly to solving difficult school-style problems (although there are exceptions). Students of ordinary schools who wish to enter colleges with more selective programs in mathematics, however, search for opportunities to prepare better for exams (traditionally, each college had its own entrance examinations; now, these examinations have been replaced by a standard exam for the entire country, the Uniform State Exam – USE). Various manuals for preparing for the USE have become an important source for such preparation in the last few years (for example, Semenov, 2008). Consider the following problem as an example:

> Among all the integers that do not constitute solutions of the inequality $(10^{4x-9}-1)(3^{5x-21}-1)\geq 0$, find the integer that is the least distance to the set of solutions of this inequality. (Semenov, 2008, p. 69)

The solution to this somewhat artificial, although not difficult, problem requires solving an exponential inequality, defining the integers that do not belong to the corresponding set, grasping the notion of the distance from a number to a set, and finally comparing numbers (fractions). Clearly, such an exercise requires a good bit of time.

In examining the content of school electives in higher grades, one must mention two books, by Sharygin (1989) and Sharygin and Golubev (1991), which brought together many difficult problems from the entire range of school mathematics, thus making these problems accessible to teachers of school electives. The problems in these books were often organized and classified in terms of the methods used for solving them. As a result, the books were not simple. However, they came to exert an evident influence on many subsequent publications. Consider the following example of a relatively easy problem from these texts:

> Given a right triangle ABC with legs AC=3 and BC=4 and two points M and K, such that MK=8, AM=1, BK=2, find the area of triangle CMK. (Sharygin, 1989, p. 167)

This problem is offered as an illustration of the notion that in a geometric problem it is important to identify the distinctive features of the figure that is given, and, in particular, the role of the numbers given. Indeed, once students start to draw the figure, they will notice that MK-AM-BK=5=BC. This means that the points M and K lie on the straight line $\overleftrightarrow{AB}$. Since it is given that MK=8, all that remains to be done is to find the length of the altitude from the vertex C to the straight line $\overleftrightarrow{AB}$, which is not difficult at all. The length of the altitude equals $\frac{12}{5}$, and the area being sought equals $\frac{48}{5}$.

On Various Forms of Distance Learning[4]

In this section, several forms of extracurricular work that take place outside specific schools are discussed (although, of course, the role of the teacher and the school in providing information about them to the students and in offering subsequent support is very important). The first activity of this kind that should be mentioned is independent reading.

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[4]. Distance education or distance learning is understood in this instance very broadly as any form of education of students that does not involve direct contact with the teacher. In recent years, this term has been frequently used to refer to education online. To repeat, here the term is used in a broader sense, if only because forms are discussed that were employed long before the internet became a part of everyday life.

Reference was made earlier to many books published specifically for schoolchildren interested in mathematics. Both in the USSR and later in Russia, numerous collections of difficult problems were published and republished, along with comparatively short and accessible presentations of various mathematical theories. In particular, certain books from the series *The Little Kvant Library* should be singled out, as well as the already mentioned pamphlets from the series *Popular Lectures in Mathematics*. Books with a more explicit and closer connection to the school course in mathematics, which are intended for an audience of many thousands or perhaps even many millions, have been and continue to be published as well. Among them are books published under the general title *Supplemental Pages for the Textbook.*

Depman and Vilenkin's (1989 and other editions) book, which is addressed to fifth and sixth graders contains, for example, the following sections:

- how people learned to count;
- the development of arithmetic and algebra;
- from the science of numbers;
- mathematical games;
- mathematics and secret codes;
- stories about geometry;
- mathematics and the peoples of our homeland; and
- how measurements were made in antiquity.

This book is precisely not a textbook. Students can (and will want to) read it at home on their own. It is written in a colloquial style and contains many historical and entertaining facts, but also includes a considerable number of problems and stories about various areas of mathematics.

Supplemental Pages for the Algebra Textbook (Pichurin, 1999), a book that addresses students in Grades 7-9, is written in a somewhat drier style, but has the same objective: to give an accessible and entertaining account of topics that comparatively strong students could have been told about in class, but which inevitably remain beyond the bounds of the school course in mathematics. The text includes stories about the evolution of algebra and several of its sections (for example, Diophantine equations or continued fractions), and generally attempts to identify key mathematical ideas as well as stages in the development of mathematics (for example, a section entitled "Turning Point in Mathematics" tells about Descartes's contribution and the appearance of the concept of variables).

Other books in the series are meant to accompany other parts of the mathematics curriculum, for example, *Supplemental Pages for the Geometry Textbook* (Semenov, 1999) and *Supplemental Pages for the Mathematics Textbook* for grades 10-11 (Vilenkin et al., 1996). The purpose of these and other books is to support students' independent reading and self-education. Thus, Pichurin (1999) concludes his book with a section entitled "Reading Is the Best Way to Learn," in which he lists various books that interested students can use to continue their mathematical education.

It might be noted, however, that although the aforementioned book by Pichurin was published in 1990 in an edition of 500,000 copies (in Russia, the size of the edition is indicated in the book), in 1998 it was reissued in an edition of only 10,000 copies. By this time, the whole system of book publishing had undergone a radical transformation. Nonetheless, independent reading remains an extremely important way for many thousands of students to become more closely acquainted with mathematics. Moreover, the limited availability of printed books is partly compensated for by the Internet: for example, the website of the Moscow Center for Continuous Mathematical Education contains a well-stocked mathematics library.

Nevertheless, regardless of how significant independent reading may be, a student often cannot manage to succeed without guidance from a teacher. Sometimes, teachers can and want to offer such help, which is all for the good, but even in the absence of such support in school, students can acquire help by taking classes at mathematics correspondence schools. As discussed in Chapter IV, mathematics correspondence schools were originally created in the 1960s under the supervision of one of the greatest Russian mathematicians, Israel Gelfand. Students who enroll in them receive pamphlets in the mail with expositions of various areas of mathematics, examples of problems with solutions, and problems to solve on their own. The students solve these problems and send them back to schools, where they are usually checked and graded by students from the universities under whose aegis the correspondence schools operate; after this, the graded homework assignments are sent back to the students. Gradually, a framework has developed in which not just individual students can enroll as students in correspondence schools, but entire classes or groups of students could do so as well (as "collective students"). Within such a framework, teachers at ordinary schools can inform and organize their students, and at the same time learn together with them and continue their own education.

Correspondence mathematics Olympiads (Vasiliev et al., 1986), which became an important form of Olympiad activity, have been mentioned already. These Olympiads are distinct because problems that were assigned to be solved over an extended period of time at home needed to differ from problems used in the ordinary Olympiads, which had to be solved on the spot. A few words should also be said about the "ordinary" assignments given in correspondence schools. To illustrate this is one assignment of the Petersburg Correspondence School (one of several centers of correspondence work that sprung up outside of Moscow).

The pamphlet *Problems in Algebra and Calculus* (Ivanov, 1995) is mainly devoted to solving problems whose formulations resemble ordinary school-style problems by using ideas from calculus and combining these ideas with standard ideas from the school curriculum. The exposition begins with an analysis of several problems and a discussion of the intermediate value theorem, which is employed in their solutions. Among the variety of problems analyzed is the following: “For what values of the parameter *a* does the equation $\sqrt{2-x}+\sqrt{2+x}=x^2+a$ have a solution?” (p. 2). The solution becomes obvious if one uses the derivative to sketch the graph of the function $y=\sqrt{2-x}+\sqrt{2+x}$ and determine its maximum. Another section of the pamphlet is devoted to the composition of functions and the concept of the inverse function. Here, a certain theory is presented (again in the form of solutions to several problems), and then different ways of utilizing it are demonstrated.

Based on the analyzed material, several problems are posed, including the following:

- Prove that the equation $\sin x = 2x+1$ has a single solution.
- How many solutions, depending on the value of *a*, does the following equation have: $\sqrt{x^2-4}=a-x^2$?
- Is it true that function f is invertible if the function $g(x)=f(x^3)$ is invertible?

The pamphlet contains 24 analyzed examples and 40 unsolved problems. Its materials form the content for two gradable assignments (15 problems each). To receive the highest grade (5), students must solve no fewer than 11 problems in each assignment, and to receive a satisfactory grade (3), they must solve no fewer than 7 problems.

The content of the pamphlet described here has a close resemblance to the curriculum of schools with an advanced course in mathematics. The topics in the pamphlets for correspondence schools, however, are varied: some pamphlets deal with traditional topics studied in ordinary schools, such as linear and piecewise linear functions, while others address topics traditionally found in mathematics Olympiads (for example, the same invariants) or still other, untraditional subjects (the title of one section in Vasiliev et al. [1986] is noteworthy in this respect: “Unusual Examples and Constructions”).

New Forms of Extracurricular Activities

Socioeconomic and technological changes have provided additional opportunities to organize mathematical competitions. Since it is not possible to describe all new forms of mathematical extracurricular activities, only the most interesting and typical of them will be considered.

Olympiad “Kangaroo”

This Olympiad emerged in Australia but came to Russia (in 1994) from France. Originally, it was offered in St. Petersburg only, but now it is available in many towns of the country, and hundreds of thousands of students participate in this Olympiad annually (making the competition very profitable financially). This Olympiad includes a multiple-choice test and three types of problems, worth 3 points, 4 points, and 5 points. All children starting from grades 3-4 are invited to participate (Bratus’ et al., 2003). Below are two 3-point problems for grades 9-10 (p. 32):

> When the flask is 30% empty it contains 30 liters more of honey then when it is 30% full. How many liters of honey does the full flask contain?

(A) 60   (B) 75   (C) 90   (D) 100   (E) 120

What is the sum $\left|2-\sqrt{5}\right|+\left|3-\sqrt{5}\right|$?

(A) $5-2\sqrt{5}$ (B) $2\sqrt{5}-5$ (C) $2\sqrt{5}+5$ (D) 1   (E) 5

Archimedes Tournament

This competition was invented by Moscow mathematics teachers for students in grades 5-6. For 5th graders, it is offered both as an individual and as a team contest; for 6th graders, it is a team contest only. School and circle teams are invited to participate. Each team has eight participants. At the individual round, 5th graders participate in a written 60-minute contest. The team round for 5th graders lasts 75 minutes and for 6th graders, 2.5 hours. This round is an oral contest. The number of participants is growing and currently hundreds of students participate; for example, in 2008, of the 553 participants, about 100 received awards (Chulkov, 2009).

Mathematical Regattas

This is another team mathematics competition. Each team should have four members. The competition has four rounds, in each of which three problems are offered for collective solving within 10-15 minutes (solutions are submitted on special pieces of paper). Teams from different towns are invited to participate. The contest is planned to serve students who are interested in mathematics but are not necessarily winners of regular Olympiads. Below, one of the three problems for the first round for 9th graders is considered (all three are to be solved in 10 minutes):

It is known that the number *a* is a solution of the equation $x^3+7x-9=0$.
Find the value of the expression $\dfrac{2a^3+3a}{11a-18}$.

(It is clear that to solve the problem, it would be sufficient to note that $2a^3+3a=18-14a+3a=18-11a$.)

Team-Individual Students' Tournament ("Mathematical All-Round")

This competition was designed for professional mathematical problem-solvers, but its form is unusual. It includes five Olympiads: three for individuals and two for teams. Individual Olympiads are thematic, either for algebra or geometry or combinatorics. The team Olympiads differ in their timing—prompt and slow. Also, some Olympiads are written and some are oral (Tikhonov & Sharich, 2012). The competition was invented and conducted for Moscow students only, but later on, teams from other cities also started to participate.

Finally, a note should be made that the competitions—some examples of which are provided—are often organized by the same people or, at least, by connected groups of people. Still, at the same time, it is accurate to observe that there are enough enthusiasts to conduct these competitions regularly.

Chapter VI

ON PROBLEMS USED IN SOME MATHEMATICAL EXTRACURRICULAR ACTIVITIES

Chapters IV and V present numerous examples of mathematical problems offered in different mathematical extracurricular activities as well as their solutions. Still, a special analysis of mathematical problems seems to be necessary, to address one of the research questions posed in this dissertation: “What topics are typically covered and what problems are typically offered to students for mathematical extracurricular activities in Russia?”

Obviously, it is not possible (nor necessary) to analyze all problems offered to students in extracurricular activities. The discussion below is limited to the problems of the district-level rounds of Moscow and St. Petersburg Olympiad (which are presented chronologically in the books by Galperin and Tolpygo, 1986, and Berlov, Ivanov, and Kokhas’, 1998).

As discussed in Chapter III, the calculations regarding the themes of the problems in different periods (and even selection of chronological periods) are not absolutely exact. For example, the same problem could be given at the Olympiads several times, that is, for different grades (in which case it was counted only once in this study), or across different years (in which case it was counted each time). Obviously, other approaches could also have been taken. Moreover, as was discussed, the identification of the theme of the problem was never entirely accurate because of the sophistication of the problems. Still, the direction of changes seems clear from the data, regardless of approach used.

Below, all district-level problems of the Moscow Olympiad before *Perestroika* (1985) and all St. Petersburg district-level problems in 1994-1998 are considered.

Moscow Olympiad Problems over 50 Years

Problems of 1935-1945

The thematic distribution of the problems of Moscow's district-level round during this time period is provided in Table 1 (see Galperin & Tolpygo, 1986).

Table 1

The thematic distribution of the problems in 1935-1945

Theme	Word Problems	Plane Geometry	3D Geometry	Numbers	Algebra	Trigonometry
Number of Problems	2	17	2	10	11	2

Most of problems in plane geometry were construction problems. One example is a problem from 1935:

> Three parallel lines are given. Construct a square such that three its vertices lie on these lines. (p. 20)

There were, however, a few problems on proving. Algebra was mainly represented by equations and systems of equations, but proofs of inequality and operations with polynomials were also present. One problem from 1941 illustrates this trend:

> Solve the equation: $|x+1|-|x|+3|x-1|-2|x-2|=x+2$. (p. 26)

Number theory problems can be typified by the following:

Add three digits to 523 . . . so that the obtained number with 6 digits would be divisible by 7, 8 and 9. (p. 25)

Problems of 1946-1955

The distribution of problems is given below in Table 2.

Table 2

The thematic distribution of the problems in 1946-1955

Theme	Word Problems	Plane Geometry	3D Geometry	Numbers	Algebra	Trigonometry	Constructions
Number of Problems	4	36	12	18	29	4	13

The increase in the total number of problems is obvious. The Olympiads of this time were conducted separately for grades 7, 8, 9 and 10. Still, even with this background, some substantive changes are visible. Among them is the fact that some new types of problems appeared, such as those represented in the column "Constructions" (which includes combinatorial problems, as discussed in Chapter III). One problem is as follows:

How many differently sized or located squares (the number of cells in which is expressed by the integer) can be drawn on the chess board with 64 cells? (p. 31)

It should be noted that even the problems listed above as belonging to traditional school areas were often not at all traditional. Here is one example that was offered to 9th-10th graders):

The 3-dimensonal body is bounded and has two distinct axes of rotation. How are the planes of symmetry of this body located? (p. 35)

Problems of 1956-1960

The distribution of problems from this time period is given below in Table 3.

Table 3

The thematic distribution of the problems in 1956-1960

Theme	Plane Geometry	3D Geometry	Numbers	Algebra	Trigonometry	Constructions
Number of Problems	26	5	22	6	4	21

Often, even problems in traditional areas such as plane geometry or algebra were unusual, and not only because of unusual ideas for a solution but because of the task itself. School mathematics curricula were under the reform at this time: new objects of study were represented, such as vectors, for example. Correspondingly, the following problem was offered:

> Inside the triangle ABC some point O is taken. Three unit vectors are constructed on the rays OA, OB, OC. Prove that their sum has a length less than 1. (p. 62)

Problems on geometrical transformation were offered, asking participants to study some unusual geometrical objects and explore inequalities and estimations, among other topics. At the same time, traditional-style problems were among those offered (although not in a great number). One such problem follows:

> Two points and a circle are given. Construct another circle that will pass the given points and will intersect the given circle so that the chord joining the points of intersection will have the given length. (p. 67)

Problems of 1961-1965

The distribution of problems for these years is given in Table 4.

Table 4

The thematic distribution of the problems in 1961-1965

Theme	Plane Geometry	3D Geometry	Numbers	Algebra	Trigonometry	Constructions
Number of Problems	36	2	18	10	1	19

Among the algebraic problems given, the following seems of interest:

> Given the quadratic equation $x^2+px+q=0$, and coefficients p and q independently take all the values from -1 through 1 (including both of them).
> Find the range of the real solutions of the equation while it happens. (p. 91)

(It is easy to find the maximal and minimal values of the solutions, but it is necessary to explain why all values from the obtained segment can be reached.)

Problems in plane geometry were diverse—from classical problems on proving to all kinds of cutting and assembling problems, and the like.

Problems of 1966-1970

The distribution of problems for this time is presented in Table 5.

Table 5

The thematic distribution of the problems in 1966-1970

Theme	Plane Geometry	3D Geometry	Numbers	Algebra	Constructions
Number of Problems	16	1	15	4	40

The following problem can serve as an example for the various problems listed under the title "Constructions":

> The Wise Cockroach can see only 1 centimeter ahead. Still Cockroach decided to search for the Truth. It is known that Truth is located in the point the distance of which to the Cockroach is D centimeters. The Wise Cockroach can make steps of length no more than 1 cm, and after each step he is informed whether he came closer to the Truth or not. Of course, he remembers everything in general and the directions of his steps in particular. Prove that he can find the Truth by making no more than $\frac{3}{2}D+7$ steps. (p. 112)

Here, one needs to offer some strategy for searching for the Truth and evaluate the number of steps it requires.

Problems of 1971-1975

The distribution of problems for these years is given below in Table 6.

Table 6

The thematic distribution of the problems in 1971-1975

Theme	Plane Geometry	3D Geometry	Numbers	Algebra	Constructions
Number of Problems	16	3	13	3	25

One example comes from 3-dimensional geometry, which was unusual for this time and area:

> All dihedral angles of the tetrahedron ABCD are acute and all opposed edges are pairwise congruent. Find the sum of cosines of all dihedral angles. (p. 122)

(The solution uses the classical ideas of school 3-dimenional geometry. The faces of the tetrahedron are congruent to each other. If one projects three of them on the fourth, the projection will make the fourth face. Meanwhile, the area of each projection can be expressed in terms of the area of the face and the cosine of the corresponding dihedral angle.)

Problems of 1975-1980

The distribution of problems for this time period is presented in Table7.

Table 7

The thematic distribution of the problems in 1975-1980

Theme	Plane Geometry	3D Geometry	Numbers	Algebra	Constructions	Analysis
Number of Problems	9	4	5	5	19	3

The following problem is counted under the heading "Analysis" (with all reservations):

> a_1, a_2,...a_n...- an increasing sequence of positive integers. It is known that for any positive integer n $a_{n+1} \leq 10a_n$. Prove that the infinite decimal $0.a_1a_2...a_n...$ obtained by

writing the terms of this sequence after the point is not periodic.
(p. 142)

Problems of 1981-1985

The distribution of problems for these years is given in Table 8.

Table 8

The thematic distribution of the problems in 1981-1985

Theme	Plane Geometry	3D Geometry	Numbers	Algebra	Constructions	Analysis
Number of Problems	26	5	16	12	32	5

The following problem represents the area of analysis:

The function $y = f(x)$ is defined on the set of all real numbers; for some number $k \neq 0$ the equality holds $f(x+k) \cdot (1 - f(x)) = 1 + f(x)$. Prove that this function is periodic. (p. 144)

At the same time, among the offered problems was the typical "old-fashioned" algebraic problem:

Solve the equation $\frac{x^3}{\sqrt{4-x^2}} + x^2 - 4 = 0$. (p. 149)

Problems of the St. Petersburg Olympiad

The distribution of problems from the district-level St. Petersburg Olympiad is given in

Table 9.

Table 9

The thematic distribution of the problems in 1994-1998

Theme	Word Problem	Plane Geometry	3-d geometry	Numbers	Algebra	Constructions	Analysis	Trigo-nometry
Number of Problems	7	20	4	18	13	47	3	1

Problems identified as the word problems are very different from the word problems typical for school textbooks. Still, given that they belong to the category of problems where some unknown quantities should be found or estimated with the help of given data, and this structure is explicit, the problems are listed as word problems. The following problem is but one example:

> Baba-Yaga and Kaschey (heroes of Russian folk-tales) collected some number of mushrooms. The number of spots on Baba-Yaga's mushrooms is 13 times more than on Kaschey's mushrooms. However, when Baba-Yaga selected her mushroom with the minimal number of spots and gave it to Kashey, the number of all spots on her mushrooms became only 8 times more than the number of spots on Kashey's mushrooms. Prove that at first Baba-Yaga had no more than 23 mushrooms. (Berlov et al., 1998, p. 16)

(This problem can be solved by writing the inequality connecting the numbers of mushrooms owned by Baba-Yaga and Kaschey. This inequality implies the required inequality.)

Three-dimensional geometry is fairly traditional. In 1994, the following problem was offered:

> The base of the pyramid SABC is a triangle ABC such that AB=17, AC=10, BC=9. The length of the altitude of the pyramid is 30, and its base is the midpoint of the segment BC. What is the area of the pyramid section such that it passes through the point A, is parallel to the line BC and divides the altitude in the ratio 4 to 1 (counting from the vertex S)? (p. 20)

Plane geometry and algebraic problems are often traditional for school mathematics as well. For example, in 1995, the following problem was offered:

> Solve in real numbers the following system of equations:
>
> $$\begin{cases} 8a^2 + 7c^2 = 16ab \\ 9b^2 + 4d^2 = 8cd \end{cases} \quad \text{(p. 39)}$$

(By adding these equations and applying classic inequalities, it is easy to conclude that the system holds if and only if all variables equal zero.)

Still, the problems listed in the column “Constructions” (including problems in logic, combinatorics, graphs, etc.) dominated.

Some Conclusions

Very few Olympiad problems were analyzed above and the analysis was not extensive. Neither the problems’ difficulty level (whatever approach was used to define it) nor ideas utilized to solve them were discussed. It was also mentioned several times earlier that the calculations regarding the themes of these problems were approximate more than accurate. However, some conclusions can still be drawn about the process of change of Olympiad problems and their selection.

First, the diversity of Olympiad problems should be mentioned. Even the very limited number of examples considered above revealed that all types of problems were offered at the Olympiads and, correspondingly, diverse knowledge as well as diverse ideas and approaches were required to solve them.

It is clear that the Olympiad problems are connected with the school curriculum. Even without previous knowledge of the reforms of the 1960s and 1970s, one can guess about them simply by observing the increased attention made to mathematical analysis at some point. Many “traditional” difficult school problems are part of the Olympiads (that is, problems posed about standard school objects and whose solutions require reasoning more or less similar to what is used in school lessons; however, school versions are different in that the reasoning is shorter and objects are considered separately rather than in combination). For example, students are supposed to know that $2xy \le x^2 + y^2$ for any real x and y, and that equality is reached only if x=y; they discussed how to write the expression $7x^2$ (or similar) as a perfect square; they used addition of the equations as a tool in solving systems. All these ideas together provide the solution to the last problem (about the system) which we considered.

Still, it is hardly possible to agree that “the important feature of the Olympiad problems is that no knowledge outside of the standard school curriculum is required for their solution” (Tikhomirov, 2008, p. 3). The authors of this statement rushed to add some corrections to it:

> Of course, this is true only approximately—some “non-school” approaches and methods as method of mathematical induction are not an obstacle for the creators of the problems since long time ago. (p. 3)

The issue is not limited to induction. To be sure, solving some problems in combinatorics or a theory of graphs does not require previous knowledge and use of theorems. Still, the experience of working with these concepts is helpful, and while the school typically does not provide this, the circle does. As a result, even district-level rounds of the Olympiads turn into a competition between circles.

When analyzing the calculations made above, one can see a few instances of the increase of the portion of problems called “Constructions.” At the same time, there evidently was no formal decision to make this change. The process of changes was not straightforward because conditions of the Olympiads were never absolutely the same (the number of problems varied almost every year) and because much depended on the people involved in writing the problems. No further generalizations seem necessary at this point, but more should be explored in the future using different approaches including interviews with members of the Olympiad Juries and analyses of Olympiad problems offered in other cities. Still, the conclusion about the increased number of “non-school” problems in the Olympiad seems sufficiently supported by the data above as well as by observations about the change in problems cited in Chapter IV.

To repeat, the writing of mathematics problems is a creative activity and, therefore, dependent on those who are doing it. By analyzing information on the authors of the problems as done, for example, in the books by Fomin (1994) or Berlov et al. (1998), one can see that one author typically gave problems in combinatorics while another gave problems in geometry. The appearance or departure of a creative author could seriously change the set. But despite that possibility, the pattern discussed above is clear.

A detailed analysis of the reasons for these changes seems of interest. As has been cited already, Fomin (1994) writes that the style of the Olympiad in Leningrad (St. Petersburg) changed at the end of the 1950s (in Moscow, it was a few years earlier, as demonstrated by the data above). He explains that "it was influenced by the fact that old 'school' topics and themes were exhausted and were widely known by students participating in circles and electives" (p. 9). It would be interesting to explore how these changes were connected with the growth of interest in discrete mathematics which happened exactly at the same time. Another possible reason for the changes could be the creation of specialized mathematics schools where many topics previously considered "unusual" became part of a regular school routine studied by everybody. This study, however, did not aim to complete this in-depth analysis.

Finally, a note should be made that the "non-school" character of the official Olympiads now to some extent is balanced by new competitions. As discussed in the previous chapter, "difficult school problems" are assuming an important place in these competitions.

Chapter VII

CONCLUSION AND RECOMMENDATIONS

This study sought to answers the following research questions:

1. What does Russian scientific literature say about the goals, objectives, and forms of extracurricular activities in Russia?
2. What is the history of the development of the Russian system of extracurricular activities within the entire system of Russian mathematics education and, even broader, within the entire system of Russian school education?
3. What topics are typically covered and what problems are typically offered to students for mathematical extracurricular activities in Russia?
4. What are the most important forms of organization utilized in the system of Russian mathematical extracurricular activities?

To answer these questions, the researcher analyzed multiple publications and archival documents. Chapters II and IV, V, and VI presented extended answers to the questions. A summary is given below.

Answers to the Research Questions: A Summary

1. What does Russian scientific literature say about the goals, objectives, and forms of extracurricular activities in Russia?

Russian scientific literature defined the goals and objectives of mathematics extracurricular activities in a variety of ways during the decades. Still, it seems appropriate to single out two major goals which were always important (although during the period of 1918-1931, rhetoric was specific). The first goal is motivating students—attempt to interest them in mathematics; the second goal is preparing the mathematically strongest students and providing them with an opportunity to deepen and enrich their mathematical education.

Along with these two major goals, a few others existed, including the ideological: extracurricular activities were viewed as opportunities (supplemental but sometimes even the most important) for political indoctrination. Soviet mathematical patriotism was meant to be formed here, while the lessons were intended to be devoted to mathematics itself. So-called *productive labor* occupied an important (although not always permanent) place in the Soviet system of values; one objective of the extracurricular activities at some periods was to reinforce a connection between mathematics and productive labor as well as with Soviet industry and agriculture.

Listing and describing different forms of mathematical extracurricular activities (which will be summarized later), Russian scientific literature raised questions about their efficiency and, even more frequently, about ways to connect extracurricular and classroom activities as well as to prepare teachers to conduct extracurricular activities. The literature emphasized that this preparation should be multifaceted and include psychological components.

2. *What is the history of the development of the Russian system of extracurricular activities within the entire system of Russian mathematics education and, even broader, within the entire system of Russian school education?*

The history of the Russian system of mathematical extracurricular activities in general was aligned with the history of the development of the system of Russian school education. In the pre-revolutionary period, extracurricular work in mathematics was relatively rare, while in the first post-revolutionary period, it became to some extent a part of the general work on "establishing connections with industry." The period of Stalin's counter-reform, which required mass preparation of qualified engineers for military industry, corresponded with increase in attention to extracurricular activities. The period of Khruschev's thaw that ushered in enthusiasm and hope brought a renewed hopeful enthusiasm for extracurricular activities as well. Meantime, the period of Brezhnev's stagnation introduced formalism and bureaucracy to the organization of extracurricular work.

At the same time, the development of extracurricular activities has some specific features. As is usual in education, the processes are fairly lengthy: starting in one historic period, they continue to develop even after the end of the period and in a changed political environment. The system of extracurricular activities allowed some freedom and unpredictability, and these characteristics were still in action to some degree, even though in other fields a stricter order was established.

Much in the development of the system was defined by internal reasons and features. The mathematical community had no other opportunity for the social engagement that was important for young and active people with high self-esteem. Extracurricular activities provided a permissible niche for such interaction. Young mathematicians were looking for the forms of work with children which would enrich their own work. Thus, new forms of mathematics circles emerged.

The extracurricular system experienced the influence of technology in a broad sense. Once printing became easier and a system of book distribution was created, mass publication of literature linked to extracurricular activities emerged. By contrast, when the book distribution system collapsed, the number of copies of publications dramatically decreased. Technological advances, from the radio to e-mail, were used in the system of extracurricular activities without changing its substantive characteristics. The content and subject matter of the extracurricular work in mathematics also developed historically, under the influence of both internal and external changes.

3. *What topics are typically covered and what problems are typically offered to students for mathematical extracurricular activities in Russia?*

As far back as the pre-revolutionary period, two major directions of mathematical activities emerged: the study of the supplemental sections of mathematics and the solving of difficult problems. (The distinction between these two directions can only be made with some reservations: systematically solving difficult problems, say, on constructions with a compass only would create theory supplemental to the regular course of study.) In turn, in the second direction, two types could be distinguished: the solving of puzzles and the solving of difficult school problems.

The supplemental sections could be borrowed from college mathematics courses; some topics could be presented earlier than in the usual course of study. Nevertheless, the source for supplemental sections was typically not that but, rather, "forgotten" theories created at some point but then excluded from the standard course in both secondary school and college. Some examples of these theories could be, for example, found in the manuals called *Beyond the Textbook's Pages*, mentioned earlier. On the contrary, sometimes relatively recent theories were delivered to the students in simplified ("non-college") versions; some of these examples can be found in the aforementioned booklets of the series *Popular Lectures in Mathematics*.

Eventually, a new group of topics was developed from the new nontraditional school sections such as logic or combinatorics. One may conclude that at some moment these themes started to dominate the Olympiads. In fact, some emerging topics became mandatory for every student striving to succeed at the Olympiads, including the pigeonhole principle, invariants, some geometric inequalities, and the like.

The process of changing the subject matter was not easy or straightforward. Some typical "supplemental" sections became part of the standard course in specialized mathematics schools. Some traditional sections of so-called higher mathematics (and higher algebra in particular) made their way down to the school course, most often to its extracurricular part (e.g., number theory problems or the theory of polynomials).

At the same time, traditional school problems—in plane geometry or on quadratic equations— are still posed and solved. The extracurricular work indicates that these sections are alive not only as steps for future education, but also as fields in which one can invent something new.

4. *What are the most important forms of organization utilized in the system of Russian mathematical extracurricular activities?*

During decades of developing mathematical extracurricular activities, a great number of forms was created. The most prominent form was the circle, which consisted of regularly organized meetings of permanent groups of students. These circles (as it was discussed in Chapter 5) could be partitioned into mass and selective with a significant difference in content between the types and with a different structure and form for the meetings as well. Competitions are the prevalent form of other extracurricular activities, and there is an extremely high number of them. They included traditional individual Olympiads and individual contests with a special system, e.g., the "Kangaroo," where students simply select an answer instead of providing an essay-style solution with all details. Also, there are team competitions and competitions that combine team and individual approaches. Finally, there are Internet competitions.

Mathematical lectures were popular for some time previously**,** but now seem to have given way to the independent reading and other forms of distance education. The Internet has provided new opportunities in this aspect. To some extent, the publication of school newsletters adhered to these forms of distant and independent learning because students could read interesting mathematical facts there. Still, school newsletters and journals seem to be more valuable for their publishers (students' editorial boards) than for their readers**.**

One more group of activities is comprised of theatrical activities. Obviously, it is not possible to count as pure mathematical work the performances (designed and performed by students and teachers) about the adventures of the brave unit that defeated the evil zero who attempted to multiply everybody by himself, as well as other exciting mathematical tales. However, one can conclude that these entertaining forms were valuable for "humanizing" mathematics and attracting students to it.

In summarizing the analysis of these different forms of extracurricular activities, it is possible to see that many of them were in fact traditional and did not change substantially over the decades (this does not exclude the appearance of new forms).

Finally, returning to the differences between elite and mass forms of work, it should be noted that they remain closely connected. Students go to the circles often because yet their parents attended the circles, and it is simply expected. Parents might attend mass circles while their child grows into a winner of an Olympiad and from there into a serious mathematician; however, it can happen within the same system. Such an example can encourage educators and researchers to become interested not only in the proverbial “tip of the extracurricular activities iceberg” but in its foundation as well.

Recommendations

A few recommendations based on the completed study are offered below. First, recommendations for future study based on the results of this study are presented, followed by a few practical thoughts and suggestions based on this exploration.

On Future Research in the Same Direction

This work is based on existing literature, including unpublished sources that survive in archives. It seems promising to supplement this study by means of oral history. In other words, one may seek answers to the following research question: What do participants in mathematical extracurricular activities (e.g., students, teachers, authors of problems) remember about them and what do they consider most important about them?

Karp (2010c) cites two cases of participants in extracurricular activities: one student was very successful and became a winner of many Olympiads, another was less successful – she found the circle activities very boring, ceased participating, and lost interest in mathematics. It would be of interest to collect more cases; while they may not provide strikingly different information from what has already been obtained, it would be helpful for a deeper understanding of the everyday life of the circle and the interests of the participants.

The voices of the participants would also allow a greater understanding of some aspects of the extracurricular activities that were not able to be more thoroughly explored in this study. In particular, an exploration of the differences between male and female participants would be extremely interesting. A quick look at the lists of Olympiad participants makes immediately evident that there are significantly more male than female winners. Thus, one might ask: At which step of a student's education does this disproportionality emerge?

Although this study attempts to use data about extracurricular activities outside of Moscow and St. Petersburg (Leningrad), the difference between "capital cities" and other locations was not sufficiently addressed. Mathematical life and the fate of the Moscow participants in extracurricular activities seem very different from the life and fate of the same-aged child in a small Siberian town, for the simple reason that lifestyle and culture differ significantly. An analysis of the "provincial" mathematical extracurricular work would be of scientific interest.

Another direction of interest is a further study of the problems used in extracurricular activities. In Chapter VI, some questions were posed that could not be addressed in this study. One such question related to the reasons for changing the Olympiad problems and the influence of research mathematics in which discrete mathematics began to attract greater attention. This is only one of a number of possible issues to explore relating to problems in extracurricular activities. It seems that a deeper analysis of these problems to include study of their solutions rather than only their themes will be of significant interest. Some guiding questions here are: Which ideas and directions of reasoning are most typical for extracurricular activities? What changes (if any) happened to the most typical strategies in problem solving over time? How is the exchange of ideas and materials between standard courses and extracurricular activities conducted? How does the understanding of what is easy and what is difficult change (if it does)? In general, a deeper analysis of mathematical challenges (Barbeau & Taylor, 2009) can and should become a subject of further study.

The study of Russian extracurricular activities is not complete without a comparison with similar work abroad. Today, when many varieties of international competitions and Olympiads take place, and, correspondingly, all forms of associations and unions of people involved in these and similar activities exist, the exchange of materials, problems, and ideas is evident. However, exchange took place and new materials found their way into the country many years ago, even when the Iron Curtain seemingly isolated the country. It would be of great interest to track the fate of these borrowed ideas and explore what happened abroad with ideas and materials that originated in Russia. Finally, it is important to explore to what extent the processes that occurred in Russia were unique (i.e., the process of changing themes of problems) as compared with the processes in other countries. Both the similarities and differences in development are of interest.

Some Practical Recommendations

The sets of problems prepared during decades of teaching are of interest not only for historical research study, but for practical use as well. A large number of mathematics educators are unfamiliar with or unaware of many of these problems. Even if these problems are translated and published outside of Russia (and many Olympiad problems are), they are typically known and used only by Olympiad participants and coaches. There is a need to organize thematic and structured groups of challenging problems so that teachers can use them in their work.

Some problems among the tasks used in extracurricular activities are very difficult and are appropriate for only a few dozen students. But among these tasks are problems that can be useful for thousands or even millions of students. Thus, it is particularly important to translate and organize these materials and make them widely available. As suggested before, the foundation of the iceberg of extracurricular activities deserves no less interest and focus than its tip.

The dissemination of the experience of mass extracurricular activities, including theatrical performances, newsletters, relatively easy competitions, and other forms seems important and useful. It is precisely in this way that interest in mathematics can be aroused in millions of students, attracting them to the subject and overcoming the fear of mathematics.

While the mathematical part—the preparation and translation of the materials for this work—is very important, of equal importance is administrative work and teacher preparation.

The Russian experience offers evidence that it is extremely important and beneficial to support teachers not only in working with slow learners, but also in working with students interested in mathematics and cultivating their interest. As discussed, the Russian system has supported those who demonstrate interest and ability in mathematics, which is perhaps one major reason for the success of Russian mathematics. It is not possible for other nations to copy the Russian system directly, nor would it be effective. However, if a society recognizes the importance of developing students' interest in mathematics, it should also be reflected in educational policy.

There one last important practical issue concerns the preparation of teachers who are able to conduct extracurricular activities. Unfortunately, prospective teachers rarely receive an opportunity to familiarize themselves with the existing treasures that have been collected internationally over decades of extracurricular activities. No courses discuss methods of conducting such work and understanding the various forms of this work, and the psychological preparation of teachers for this work is barely considered.

Often the underlying reason for such a lack of discussion is the lack of confidence in the ability of both teachers and students. The Russian experience clearly demonstrates that teachers on a mass scale can conduct extracurricular activities if they are supported by the administration and if they receive appropriate professional help.

In conclusion, there is a need to develop special courses for teachers and to publish special manuals for teachers to assist them in conducting extracurricular mathematical activities. It is hoped that the Russian experience presented in this dissertation can guide that process.

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